A
MATHEMATICAL
MISCELLANY,
IN FOUR
R
PARTS.

I. An **ESSAY** towards the probable SOLUTION of the Forty-five ſurprizing PARADOXES in *GORDON*'s Geography.

II. Fifty-five new and amazing PARADOXES, ſome in VERSE, ſome in PROSE, with their SOLUTIONS.

III. An *Algebraical* Solution to the Hundred *Arithmetical* and *Geometrical* PROBLEMS, left unanſwered in *HILL*'s *Arithmetick*, and *ALEXANDER*'s *Algebra*: In the Solution of which, the Young *Algebraiſt* will find ſuch a Variety of Examples, performed after ſo conciſe and plain a Method, as will enable him to comprehend the moſt abſtruſe Parts of that ſublime SCIENCE.

IV. Miſcellaneous RULES about forming *Ænigmas*, *Queſtions*, the Doctrine of *Eclipſes*, of *Pendulums*, the *Equation* of *Time*, concerning *Eaſter*, &c.

By a Lover of the MATHEMATICKS.

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BRITISH MUSEUM

PART. I.

GORDON's PARADOXES SOLVED.

A PARADOX is a *ſeeming* Falſity, but a *real* Truth; 'tis that, which to *unthinking* Perſons, ſeems abſurd or impoſſible; but to a *thoughtful* Man, is plain and evident: The main Drift whereof is to *whet* the Appetite of an *inquiſitive* Learner, and to ſet him upon *Thinking*.

1. *There are two remarkable Places on the* Globe *of the Earth, in which there is only one Day and one Night throughout the whole Year.*

Anſwer. The two remarkable Places are the two *Poles*; for to the *North Pole* the *Sun* riſes about the 10th of *March*, and ſets not till about the 12th of *September*, and the enſuing Twilight continues till the *Sun* be eighteen Degrees below the *Horizon.* i. e. about the 2d of *November*, then dark Night continues till about the 18th of *January*, at which Time the Day breaks, and the Morning Twilight continues till *Sun* riſe on the 10th of *March.* Hence 'twixt *Sun* riſe and *Sun* ſet, are ſix Months, but betwixt Day-break and Twilight's End, are about two hundred and eighty eight Days, but totally dark only ſeventy ſeven Days.

Note, When it riſes to the *North Pole* it ſets to the *South*, *&* *e contra*; and becauſe it riſes but once and ſets but once in a Year, to either, there is but one Day and one Night in the whole Year.

2. *There are alſo ſome Places on the Earth in which 'tis neither Day nor Night at a certain Time of the Year, for the Space of twenty four Hours.*

Anſwer. If by *neither Day nor Night*, be meant Twilight, it may be any Climate of the *Frigid Zones*: but if it be underſtood that the *Sun* neither riſes nor ſets for 24 Hours, the Places muſt be ninety Degrees diſtant from the *Sun:* Thus, if the *Sun* be in the *Æquator*, then the *Poles* are the Places; for at thoſe Times the *Sun* circuits about their *Horizon* for twenty-four Hours, half above and half under it; hence for ſo long, 'tis neither Day nor Night then and there: Thus alſo, if the *Sun* were in the *Tropic* of *Capricorn*, ninety Degrees from which would be the *Artic Circle*, when and where the *Sun* would be neither above nor below their *Horizon* for twenty-four Hours.

3. *There is a certain Place of the Earth, at which, if two Men ſhould chance to meet, one would ſtand upright upon the Soles of the other's Feet, and neither of them ſhould feel the other's Weight, and yet they both ſhould retain their natural Poſture.*

Anſwer. He ſays *of the Earth*, not *on the Earth*, which therefore means the Center thereof; for imagine an Hole board thro' from our Feet, to and thro' the Center of the Earth, to the oppoſite Point, or the *Antipodes*, and one Man deſcended toward the Center at one End of the Hole, and another Man deſcended at the other End of the Hole, till they both met at the Center, ſo would they ſtand on each other's Feet, with their Heads towards the *Zenith*, in their natural Poſture without feeling each other's Weight. According to theſe Maxims, *No heavy Body gravitates in the Center*, and *All heavy Bodies tend to the Center*; whereas a *Gravitation* at the very Center, muſt imply neceſſarily a *Divergency* from the Center, or an Aſcent, which is abſurd.

4. *There is a certain Place of the Earth, where a Fire being made, neither Flame nor Smoke would aſcend, but move circularly about the Fire: Moreover, if in that Place one ſhould fix a ſmooth or plain Table, without any Ledges whatſoever, and pour thereon a large Quantity of Water, not one Drop thereof would run over the ſaid Table, but would raiſe itſelf up in an Heap.*

Anſwer. This Place muſt alſo be the Center of the Earth, for the Reaſons mentioned in the laſt.

5. *There is a certain Place on the* Globe, *of a conſiderable* Southern *Latitude, that hath both the greateſt and leaſt Degree of Longitude.*

Anſwer. This may either mean the *South Pole*, which hath not only the leaſt but greateſt, and all intermediate Degrees of Longitude, all which meet in the *Poles*. Or all Places that lye under the firſt *Meridian*, have both the leaſt and greateſt Degree of Longitude, as at Noon is the greateſt and leaſt Number of Hours; becauſe then and there we begin and end our Reckoning.

Thus *Trinidada* is in Latitude South, 20 Degrees, and under the firſt *Meridian*, with them who reckon from St. *Michaels*'s, or *Triſtam de Cunha*, is in thirty-ſix Degrees fifty four Minutes *South* Latitude, and ſame Longitude, with *Teneriff*; all which, according to the old Way of reckoning Longitude, are not only in the Beginning of the firſt Degree, but alſo in the End of the three hundred and ſixtieth Degree of Longitude.

6. *There are three remarkable Places on the* Globe *that differ both in Longitude and Latitude, and yet all lye under one and the ſame* Meridian.

Anſwer. By the *Globe* may be meant the *Artificial Globe*, and by the *Meridian* may be meant the *Brazen Meridian* belonging to it, which may be five Degrees thick; then ſuppoſe three Places, *A*, *B*, *C*, *A* to be *Dublin*, in Latitude fifty three Degrees and an half, and Longitude twenty Degrees; *B* to be *Liſbon*, in Latitude thirty eight Degrees and an half, Longitude eighteen Degrees; *C* to be the Iſle *Paxaros*, in Latitude eight Degrees, and Longitude two Hundred Degrees; all which, tho' they differ both in Longitude and Latitude, yet may they all lye under one and the ſame *Brazen Meridian.*

Or without Equivocation, ſuppoſe one Place under the *Pole*, a ſecond on this Side, and a third on the other, under the ſame *Meridian* Circle, ſo may they all differ both in Longitude and Latitude; for the *Pole* contains all Degrees of Longitude.

7. *There are three remarkable Places on the* Continent *of* Europe, *that lye under three different* Meridians, *and yet all agree both in Longitude and Latitude.*

Anſwer. Divers *Geographers* begin their firſt *Meridian* at divers Places; thus, *Ptolomy* at *Cape Verde* (formerly one of the *Fortunate Iſlands*) *Mercator* at *St Michael*'s, in the *Azores*; *Blaeu* at *Teneriff*, one of the *Canary Iſles*, &c. Now if you take (under the ſame Latitude) three Places (ſuppoſe ten Degrees from each of theſe firſt *Meridians*) they agree all in Latitude, alſo in Longitude, from theſe three reſpective Places, and yet lye under three different *Meridians* in reſpect of the *Globe*; or which is much to the ſame Purpoſe, chuſe any three Places under different *Meridians*, and one Parallel of Latitude, as *Pico*, St. *Nicholas* and *Liſbon*, and begin the Longitude at every one of them; ſo will they all be firſt *Meridians* and agree in having no Longitude, and being in the ſame Parallel will agree alſo in Latitude.

8. *There is a certain Iſland in the Ægean Sea, upon which if two Children were brought forth at the ſame Inſtant of Time, and living together for ſeveral Years, ſhould both expire on the ſame Day, yea, at the ſame Hour and Minute of the Day, yet the Life of one would ſurpaſs the Life of the other by divers Months.*

Anſwer. Different Parts of the ſaid Iſland may be ſuppoſed to compute differently, ſome by the *Solar* Year, ſome by the *Lunar*; or ſome uſe different Courſes of the *Moon*; in one Place her *Periodical*, and in other Places her *Synodical*, which is a larger Revolution than the former; ſo within the Compaſs of ſome Years the Difference will amount to ſeveral Months: Or ſuppoſe the Iſland to be *Negropont*, in the *Ægean* Sea, where both *Chriſtians* and *Turks* dwell; now the *Turks* follow the *Lunar* Year, which is 11 Days leſs than the *Solar*, which the *Chriſtians* account by. Now, if the Children ſhould live thirty *Solar* Years together, and then die, the *Turks* would account them about ten Months older than the *Chriſtians*. Or if one of the Children ſails directly *Eaſt*, and the other directly *Weſt*, when they encompaſs the *Globe* of the Earth, once (which is now eaſily done in a Year) there will be two Days Difference in their Age, and in forty Years thus ſailing, the one would be eighty Days older than the other. Or ſuppoſe the one lives without the *Artic Circle*, where no Day exceeds 24 Hours, and the other goes and lives in the Latitude of 73 Degrees 20 Minutes, where the Day is three Months long, and then returns, and both die at one Inſtant, the one will be three Months older than the other; but the two firſt Solutions ſeem preferable, becauſe of theſe Words in the Paradox, *living together for ſeveral Years*; yet becauſe the two laſt carry Inſtruction along with them, I would not omit the Mention of them.

9. *There are two obſervable Places belonging to* ASIA *that lye under the ſame* Meridian, *and of a ſmall Diſtance from one another; and yet the reſpective Inhabitants of them, in reckoning their Time, do differ an intire natural Day every Week.*

Anſwer. This alſo may be ſolved two Ways, firſt, if they keep their Sabbaths on different Days of the Week, as the *Chriſtians* on *Sunday*, the *Græcians* on *Monday*, the *Perſians* on *Tueſday*, the *Aſſyrians* on *Wedneſday*, the *Ægyptians* on *Thurſday*, the *Turks* on *Friday*, and the *Jews* on *Saturday*. Or better thus, the two Places, are one *Macao*, and the other the *Philippine Iſles*, near each other, and under the ſame *Meridian*, yet they differ one Day in their Account; for in the *Philipine Iſles* the *Spaniards*, when it is their laſt *Saturday* in *Lent*, the *Portugueſe* in *Macao* eat Fleſh, it being their firſt *Sunday* in *Eaſter*: The Cauſe of this Difference is the *Spaniards* ſailed thither *Weſterly*, and loſt half a Day, and the *Portugueſe* ſailed thither *Eaſterly* and gained half a Day.

10. *There is a particular Place of the Earth where the Winds (tho' frequently veering round the* Compaſs) *do always blow from the* North *Point*.

Anſwer. Under the *South Pole* directly; for all Winds blowing there muſt needs blow *North*, as all Winds blowing at the *North Pole* muſt needs blow *South*, becauſe there the *Meridians*, which are *North* and *South*, are the *Azimuths* all concentring in the *Pole*, which is their *Zenith*.

11. *There is a certain Hill in the South of* BOHEMIA, *on whoſe Top, if an Equinoctial Sun-Dial be duly erected, a Man that is Stone blind may know the Hour of the Day by the ſame, if the* Sun *ſhine*.

Anſwer. Perhaps it never doth ſhine on that Hill becauſe there is a Mountain that is ſaid to incircle all *Bohemia*, or never till Noon; ſo then if you tell the blind Man the *Sun* ſhines, he'll tell you 'tis Twelve o'Clock: Howbeit, I know not a better Way to make a blind Man's Sun-Dial than this.

Fill a Glaſs Globe with Water, which fix in a Sphere, with twelve poliſhed Iron Meridians, each having ſo many Nicks as the Number of Hours belonging thereto; which let be fixed preciſely at the Diſtance of the *Focus* from the *Globe*, ſo will the *Globe* full of Water unite the ſolar Rays that they'll burn at a Diſtance; thus the equinoctial Dial being fix'd in the Sun-ſhine, on a Hill or Valley, one that is Stone blind may feel which *Meridian* is hotteſt, and grope out by the Nicks the Number of the preſent Hour.

12. *There is a conſiderable Number of Places lying within the* Torrid-Zone, *in any of which, if a certain kind of* Sun-Dial *be duly erected, the Shadow will go back ſeveral Degrees upon the ſame at a certain Time of the Year, and that twice every Day for the Space of divers Weeks; yet no ways derogating from that miraculous returning of the Shadow upon the* Dial *of* Ahaz, *in the Days of King* Hezekiah.

Anſwer. Any-where in the *Torrid-Zone*, where the Latitude is leſs than the Declination of the *Sun*, and both towards the ſame *Pole:* The *Sun* comes twice to the ſame Point of the *Compaſs* both Forenoon and Afternoon; and an equinoctial Dial placed horizontally, the Shadow of the Gnomon ſhall go back, *plus minus*, twice every Day. But becauſe the Paradox mentions *a certain kind of Dial*, I ſuppoſe it may be thus anſwered, by a plain equinoctial Dial, deſcribed on both Sides of a Horizontal Plain, and with two Gnomons, and near the *Tropic*, when the Latitude and Declination are equal; before the *Sun* comes to the mathematical *Horizon* in the Morning, he will ſhine on the lower Side of the Plain, and the Shadow of the Gnomon will run *Weſtward ad infinitum*, and preſently after ſix o'Clock, as he ſhines on the upper Plain, the Shadow runs *Eaſtward* till Noon, and thence to ſix in the Evening, at which Time the Shadow on the lower Plain will begin and run *Weſtward* till Sun-ſet; there may, by concave, convex and reflect Dials, be other Ways of ſolving this.

Note, In the Latitude 18 Degrees *North*, the Retrogradation of the Shade will continue, more or leſs, from the firſt of *May* to the 26th of *July*, *i. e.* 80 Days, which the *Sun* ſpends in moving from 18 Degrees *North* Declination, till it come back to the ſame Degree again.

13. *There are divers Places on the* Continent *of* AFRICA, *and the Iſlands of* Sumatra *and* Borneo, *where a certain kind of*

Sun-Dial *being duly fix'd, the Gnomon thereof will caſt no Shadow at all during ſeveral Seaſons of the Year; and yet the exact Time of the Day be known thereby.*

Anſwer. A Horizontal Dial under the Equinoctial Line caſts no Shadow at twelve o'Clock, twice every Year; or becauſe the Places mentioned in the Paradox are betwixt the *Tropics*, the *Sun* comes twice in the Year to their *Zenith*, and then the Gnomon caſts no Shadow exactly at Noon; or it may be the blind Man's Dial aforeſaid in the eleventh Paradox: But I rather take it to be a *Globe* rectified according to the Latitude and Day of the Month, and the Index to the Hour Twelve, and to the *Sun*'s Place apply a Perpendicular or Spheric Gnomon, which is to be there fixed, and the Globe turned till it caſt no Shadow, ſo will the Globe's Index point out the Hour any Time when the *Sun* ſhines as well in theſe Parts of the World as in thoſe Iſlands; for Dials may be made to ſhew the Hour, without Shade of either Stile or Hour-Line, as a Globical Dial, having a moveable Equator, and a fixed Stile or Gnomon thereon.

14. *There is a certain Iſland in the vaſt* Atalantic Ocean, *which being deſcryed by a Ship at Sea, and bearing due* Eaſt *off the ſaid Ship, at twelve Leagues Diſtance by Eſtimation;*

tion; the trueſt Courſe for hitting the ſaid Iſland is to ſteer ſix Leagues due Eaſt, *and juſt as many due* Weſt.

Anſwer. The Prime *Meridian* from whene Longitude is accounted, both Ways, *Eaſt* and *Weſt*, paſſes thro' the Middle, betwixt the Ship and Iſland, and ſo Regard is had to the *Eaſt* and *Weſt* Longitude, and not to the Ponts of the *Compaſs*.

15. *There is a remarkable Place in the* Globe *of the Earth, of a very pure and wholſom Air to breathe in; yet of ſuch a ſtrange and deteſtable Quality, that it is abſolutely impoſſible for two of the intireſt Friends that ever breathed to continue in the ſame in mutual Love and Friendſhip for the Space of two Minutes of Time.*

Anſwer. 'Tis impoſſible for two Perſons to be in one and the ſelf ſame individual Place together: Others ſay, by Reaſon of the Earth's Motion they cannot continue in the ſame Air; others ſay, two in the Throne cannot continue in mutual Love and Friendſhip; or there may be Vulcanos, Caves and Lakes which emit ſulphureous, peſtilential and killing Vapours, tho' ſituate in a pure wholſom Air; as *Sicily*, *Judea*, and *Iceland*, in which are an *Ætna* or dead Sea, and in the laſt both; but I rather take it to be directly under the *Poles*, which by reaſon of its ſuperlative Cold, muſt needs have a pure Air; but we hear of none that ever got within an hundred Leagues of them, let alone to ſtay two Minutes there, by reaſon of the Mountains of Ice, frozen Sea, and exceſs of Cold for a thouſand Miles round them.

16. *There is a certain noted Place, in the vaſt* Atlantick *Ocean, where a briſk Levant is abſolutely the beſt Wind for a Ship that is to ſhape a due* Eaſt *Courſe, and yet ſhe ſhall ſtill go before it.*

Anſwer. If the Place be *Eaſtward* of the *Levant*, a Ship may be carried by an *Eaſt* Wind round the *Globe* to it, provided ſome *Eaſt* Point be fixed; or where there is a violent Tide; as the Gulf of *Florida* may be meant: But the Quibble may lie in the Word *Shape*; for an *Eaſt* Wind may be beſt for carrying her out of an Harbour, to ſail to a Place on the *Eaſtern* Part of that Iſland or Continent; or it may mean the Streights of *Gibraltar*; for a briſk Levant raiſes the *Mediterranean*, inſomuch that the Paſſage thro' them is the ſafer, as 'tis to come into an Harbour, when High Water: Thus, if I miſtake not, in the *Eaſt-India* Voyages, near or on the Line, a Wind from the Levant ſeems to be the only Wind to keep a Ship from being driven to the *African* Shore.

17. *There are divers remarkable Places upon the* Terraqueous Globe, *whoſe ſenſible* Horizon *is commonly fair and ſerene; and yet 'tis impoſſible to diſtinguiſh properly in it any one of the intermediate Points of the* Compaſs, *nay or ſo much as two of the four* Cardinals *themſelves.*

Anſwer. Under either of the *Poles*, in which all the Points of the *Compaſs* meet in a Center, as aforeſaid, in Paradox 5, 10, foregoing.

18. *There is a certain Iſland in the* Baltick *Sea, to whoſe Inhabitants the Body of the* Sun *is clearly viſible, in the Morning before he Riſeth, and likewiſe in the Evening after he is Set.*

Anſwer. This is occaſion'd by Refraction, ſome of whoſe Properties are, 1. That oblique Rays out of a thinner Medium, falling on a groſſer are refracted, or accede towards the Perpendicular let fall from their Entrance, But 2*dly*, Rays out of a groſſer Medium into a thinner, recede further from the ſaid Perpendicular. 3*dly*, Perpendicular Rays are not refracted. 4*thly*, The greater is the Refraction, the further the Rays are from being perpendicular. Hence 'tis that the Sun ~~and~~ Stars Refraction is greater, the nearer they be to the *Horizon*, alſo the thicker the Atmoſphere, the greater the Refraction;

as near the *Poles* and in the *Northern* Seas as the *Baltic*. Refraction is uſeful in the firſt Diſcovery of Land upon Sea, raiſing the Tops of Mountains in the Air, to be ſeen ſeveral Leagues further off, than they would be, were there no Refraction. This made the *Hollanders* in *Nova Zembla*, ſee the Sun riſe ſome ſixteen Days before they expected it, or would have ſeen it, had there been no Refraction. This alſo is the Reaſon why a Piece of Silver, which could not be ſeen in a Baſon, yet being covered with Water, a thicker Medium than Air, appears viſible: And alſo, why, if a Man, would ſhoot a Salmon under Water, he muſt not aim at that Point of the Water where the Ray of Sight enters, but a great Way on this Side of it; as, if the Salmon was in the Perpendicular let fall from the Point where the Ray of Sight enters the Water.

The ingenious *Lowthorp*, about 1700, gave occular Demonſtration of the Refraction, by making a Vacuum between two inclin'd Planes of Glaſs, by the Help of Quicſilver, thro' which an Object view'd with a Teleſcope was ſeen, upon Re-admiſſion of the Air, very ſenſibly to change Place according to the different Denſity thereof, See *P. Tranſ.* N° 257.

19. *There is a certain Village in the Kingdom of* NAPLES, *ſituated in a very low Valley; and yet the* Sun *is nearer to the Inhabitants thereof, every Noon by three thouſand Miles and upwards, than when he either riſeth or ſetteth to thoſe of the ſaid Village.*

Anſwer. The *Sun* is nearer at Noon to the Inhabitants of any Part of the Earth, as well as *Naples*, by the Semi-diameter of the *Earth*; which, by the moſt accurate Obſervations yet made, is three thouſand ſix hundred and ninety-two *Engliſh* Miles.

20. *There is a certain Village in the South of* Great-Britain, *to whoſe Inhabitants the Body of the* Sun *is leſs viſible about the Winter Solſtice, than to thoſe who reſide upon the Iſland of* Iceland.

Anſwer. This Village, perhaps, is near *Lewis* in *Suſſex*, lying under an high Mountain; there, about the Time of the Winter Solſtice, the Sun is but a ſmall Time viſible to the Inhabitants; or ſome Valley or Glin ſurrounded with Hills in or near *Wales*, that in the Winter 'tis near Noon e're the Sun approach them, and then diſappears preſently; whereas in an open Place in *Iceland*, or on the Top of an high Mountain, there the Sun may much ſooner and much longer appear, and the more becauſe of the Greatneſs of Refraction, which the Groſſneſs of the At-

Atmoſphere magnifies: Moreover, in Glins, near Mountains, 'tis obſervable, there is leaſt Sun-ſhine, becauſe Miſts are more frequent.

21. *There is a vaſt Country in* ÆTHIOPIA SUPERIOR, *to whoſe Inhabitants the Body of the* Moon *doth always appear to be moſt enlightned when ſhe is leaſt enlightned, and to be leaſt, when moſt.*

Anſwer. "The Light that falls upon "any Body, being always in a reciprocal "duplicate Ratio of the Diſtance from the "luminous Bodies." Hence it follows, that not only in *Æthiopia*, but in all Parts of the World, the Moon doth always appear to be moſt enlightned at the Full, when ſhe's leaſt enlightned, becauſe ſhe is then removed from the Sun farther than at the new Moon; by the Diameter of the Moon's Orbit, at which Time, tho' neareſt to the Sun, ſhe appears leaſt enlightned to us, when ſhe is, in Reality, moſt: Or *Æthiopia Interior*, or *Superior*, being ſituated near and under the *Tropic* of *Capricorn*, and hath Valleys ſurrounded with prodigious high Mountains, and the Terms, *moſt and leaſt enligtned* may either reſpect the Moon's Body, or the Time of her ſhining.

1. *If her Body*, obſerve, that the Moon is as well enlightned by the Earth as the Earth

by the Moon, is diſcovered by Teleſcopes in the Hands of modern Philoſophers; and neither the Ancients did, nor Moderns do queſtion, that both receive Light from the Sun: Wherefore, at the Full, when ſhe ſeem moſt enlightned to any one Place, ſhe is leaſt, in reſpect of herſelf, becauſe then, ſhe receives only thoſe Rays that come directly from the Sun: But at the new Moon, when ſhe ſeems leaſt enlightned to us, ſhe is moſt in reſpect of herſelf, becauſe ſhe receives Light from the Sun on that Side next him, and Light from his Beams reflected from the Earth to that Part of the Moon next us; ſo at Conjunction, ſhe is in a manner, wholly illuminated in herſelf, and but half in Oppoſition.

2. *If it reſpect the Time of her Shining*, ſeeing 'tis Winter in *Æthiopia* when Summer with us, *& e contra*, they have longeſt Nights when ours are ſhorteſt, and the contrary; wherefore to them the Moon will be moſt enlightned, or ſhine longeſt when to us leaſt, and alſo moſt to us when leaſt to them.

22. *There is a certain Iſland (whereof Mention is made by ſome of our lateſt Geographers) whoſe Inhabitants cannot properly be reckoned either Male or Female, nor altogether Hermaphrodites; yet ſuch is their peculiar Quality,*

Quality, that they are ſeldom liable unto either Hunger or Thirſt, Cold or Heat, Joy or Sorrow, Hopes or Fears, or any ſuch of the common Attendants of Life.

Anſwer. If Puppets, Inſects, Stones Animalcules, Birds, Fiſhes or Plants, by a Catachreſis may be called Inhabitants, then may our Author mean the Iſland of Parrots, ſituate in *Terra Auſtralis incognita*, or any uninhabited Iſland, diſcovered by our lateſt Travellers, where no other Inhabitants are, ſave ſuch Plants, Fiſh, Stones, Inſects and Animalcules.

23. *There is a remarkable Place of the Earth, of a conſiderable Southern Latitude, from whoſe Meridian the Sun removeth not for ſeveral Days, at a certain Time of the Year.*

Anſwer. 1. 'Tis doubtleſs under the *South Pole*; but, 2. Taking Sun for Sun-ſhine, by a Metonymy, it may intend any Place beyond the *Antartic Circle*; and then it will not mean that the Sun ſtands ſtill in the Meridian, but that he enlightens it for as many Days as he is above their Horizon; and this is uſual, when we ſay the Sun moves not from ſuch a Wall or Dial for ſo many Hours: Thus, in Latitude 68 Degrees South, the Sun ſhines upon its Meridian conſtantly for 30 Days.

24. *There is a certain Place of the Earth of a conſiderable Northern Latitude, where tho' the Days and Nights (even when ſhorteſt) do conſiſt of ſeveral Hours; yet in that Place it's Mid-day, or Noon, every Quarter of an Hour.*

Anſwer. Under the *North Pole*, for there every *Azimuth* is a *Meridian*, and the Sun's Courſe is nearly parallel to the Horizon all the Year.

25. *There are divers Places on the Globe of the Earth, where the Sun and Moon, yea and all the Planets, do actually riſe and ſet according to their various Motions, but never any of the fix'd Stars.*

Anſwer. Under the *Poles*, the Planets by their Motions, get North and South Declination, conſequently riſe and ſet, with relation to thoſe two Places; but the fixed Stars keeping an exact Diſtance from the *Pole*, may be ſaid never to riſe or ſet, tho' their Motion on the *Poles* of the *Eclipic*, may be thought ſome ſmall Objection to this Paradox: Or if by *divers Places*, be meant many, or more than two, he may intend any Place in *England*, *Denmark*, or *Germany*, where Spheres are, or Pieces of artificial Clock-work, *&c.* ſhewing the riſing and ſetting of the Planets, but none of the fix'd Stars.

26. *There is a very remarkable Place upon the Terraqeous Globe, where all the Planets, notwithſtanding their different Motions and various Aſpects, do always bear upon one and the ſame Point of the Compaſs.*

Anſwer. Under either of the *Poles*, for Reaſons in Paradox 5, 10, 17, 23, 24, 25; for to an Eye, ſituate in the North or South *Pole*, all the Stars however ſituate, will bear on the South or North Point of the Compaſs, becauſe every Azimuth becomes a Meridian, the Zenith and Pole being but one and the ſame Point.

27. *There is a certain noted Part of the Earth, where the Sun and Moon* (ipſo tempore plenilunii) *may both happen to riſe at the ſame Inſtant of Time and upon the ſame Point of the Compaſs.*

Anſwer. Under the *Poles*, for Reaſons in the laſt, to which add Refraction, which raiſes the Object into and above the *Horizon*, when it is conſiderably under the ſame. See Paradox 18.

28. *There is a certain Place on the Continent of* Europe, *where, if ſeveral of the ableſt Aſtronomers (the World now affords) ſhould nicely obſerve the celeſtial Bodies, and that at the ſame Inſtant of Time, yet the planetary Phaſes*

Phaſes and their various Aſpects, would be really different to each of them.

Anſwer. Some ſay, that neither at the Center, or any Part of the Earth, no one can obſerve all the celeſtial Bodies at one and the ſame inſtant of Time. 2. Others anſwer thus, if one of theſe able Aſtronomers ſhall nicely obſerve the Heliocentric Places of the Planets, another the Geocentric; their Phaſes and Aſpects would be really different to each of them. Or, 3. This Paradox may reſpect the various Syſtems of the ableſt Aſtronomers, whether *Ptolomy*, *Pythagoras* or *Tycho*, to each of whom the Planets would have both different Phaſes and Aſpects, were they nicely obſerved in any Part of the World; becauſe *Ptolomy* fix'd the Earth in the Center, the Sun betwixt *Venus* and *Mars*; but *Pythagoras* and *Copernicus* place the Sun in the Center of all, and the Earth betwixt *Venus* and *Mars*, and *Tycho* blending both Syſtems aforeſaid, borrows from each, but agrees with neither; he ſuppoſing the Earth in the Center of the Sun, Moon, and fixed Stars, daily to revolve on its *Axis*, and the Sun in the Center of the other five Planets, revolving about the Earth in one Year. Or 4. By the Word *Aſpects* in this Prradox, Reſpect may be had to the ableſt Aſtrologers quartering and triſecting, or various Ways of erecting their Schemes of the Heavens.

						th				*th*	
A	following	*Regiomontanus*	may put	*Saturn*	in the	11	and	*Venus*	in the	9	Houſe.
B		*Campanus,*		*Saturn*		12		*Venus*		9	
C		*Alcabitius,*		*Saturn*		10		*Venus*		8	
D		*Ptolomy,*		*Saturn*		10		*Venus*		8	

Thus may *B* have a ſquare maundane Aſpect of *Saturn* and *Venus*, when *A*, *C*, *D*, have a Sextile, *& ſic de cæteris.* And this may be in any Place where ſuch Aſtrologers of different Judgments meet, whether they be Deſciples of *Regiomontanus*, *Ptolomy*, &c.

Laſtly, If *celeſtial Bodies* mean, by a Metonymia *ſigni pro re ſignata*, the Planets, in certain Spheres in *Germany*, view'd by ſeveral able Aſtronomers, at the ſame inſtant of Time their planetary Phaſes and Aſpects would be really different to each of them, by Reaſon of their Parallax of Sight and Situation, one ſeeing ſome Point of the ſame Planet hid from the Sight of the other, and on a different Point of the Compaſs.

29. *There is a large and famous Country on the Continent of* Africa, *many of whoſe Inhabitants are born perfectly deaf, and others Stone blind, and continue ſo during their whole Lives, and yet ſuch is the amazing Faculty of thoſe Perſons, that the Deaf are as capable to judge of Sounds as thoſe that hear, and the Blind of Colours as thoſe that See.*

Anſwer. The Blind and Deaf have Capacity to judge of Colours and Sounds, as well as thoſe that See and Hear; tho' they want the Senſes of Seeing and Hearing: Or becauſe the Paradox refers us to the *Continent* of *Africa*, perhaps none of them have any Judgment in Colours or Sounds, as may appear by their harſh Jargon in Speech and Muſick, and profound Ignorance and Stupidity in any Thing that is curious; as in *Æthiopia* Exterior, there are Cannibals, which are ſo extreme naſty and brutiſh, that they have nothing, ſave the Shape of Men, to lay Claim to the Character of rational Creatures: They ſmear themſelves with ſtinking Greaſe, their Cloak is a Sheep's Skin juſt flead, and their Ribbons and Stockings are the Guts which they frequently feed on, as well as Human Fleſh; yea, and themſelves, tho' ſo brutiſh and ſwiniſh ſerve, as good Pork to the *Cabona*'s, a worſe ſort of Cannibals, if poſſible.

30. *There are certain People in* South America, *who are properly furniſh'd with only one of the five Senſes.* (i. e.) *that of Touching; and yet they can both hear, ſee, taſte, and ſmell, and that as nicely as we* Europeans *who have all the five.*

Anſwer. All the Senſes are properly by the Touch: In Seeing, the Object touches the *Retina*; in Hearing, the Sound touches the *Drum* of the Ear; in Smelling, the Effluvia's touch the *Senſorium*; in Taſting, the *Palate*, *&c.* Or tho' they may have them, yet (as in the laſt Paradox) they being ſo bruitiſh, and not knowing them, neither the right Uſe or Exerciſe of them, they may be ſaid, not to be properly furniſh'd with them; like us *when aſleep*, yet when taught they can uſe them as nicely, as we *when awake*; ſo they have them *in potentia*, as a Child, yet not *in actua*, as when grown up before they be taught by others.

31. *There is a certain Country in* South America, *many of whoſe ſavage Inhabitants, are ſuch unheard of Cannibals, that they not only feed upon human Fleſh, but alſo ſome of them do actually eat themſelves, and yet they commonly ſurvive that ſtrange Repaſt.*

Anſwer. If they don't eat *(their Meat)* themſelves, who can eat for them, in ſuch manner as to ſuſtain their Life; or they may be ſuch Brutes and Cannibals as to eat their Wives, Huſbands or Children, which may be ſaid to be Part of themſelves, as being Fleſh and proceeding from them.

32. *There is a remarkable River on the* Continent *of* Europe, *over which there is a Bridge, of ſuch a Breadth, that above three thouſand Men, a-breaſt, may paſs along upon the*

the ſame, without crowding one another in the leaſt.

Anſwer. The River *Guadiana*, betwixt *Andaluſia* and *Portugal*, formerly call'd *Anas*, hides it ſelf wholly at the Town *Medelina*, and about thirty-two Miles Diſtance ſhews it ſelf again; and *Alpheus*, a River of *Achaia*, runs under the Ground and Sea all the Way to *Sicily*, where, the *Grecians* ſay, it riſes again, and is called *Arethuſa*; becauſe every fifth Year, it caſts out the Dung of the Cattle that was thrown into *Alpheus*, at the Time of the Olympic Sacrifices; therefore the Land, over either of them, may not improperly be called a Bridge: Alſo in the County of *Warwick* there is ſuch a Bridge: On a Common near *Over Ichington*, is a Pool whoſe Stream entreth the Ground, and after an intricate Paſſage, of half a Mile, cometh out again and paſſeth along the Brook.

33. *There is a large and ſpacious Plain, in a certain Country of* Aſia, *able to contain ſix hundred thouſand drawn up into Battle array; which Number of Men being actually brought thither, and there drawn up, it were abſolutely impoſſible for any more than one ſingle Perſon to ſtand upright upon the ſaid Plain.*

Anſwer.

Anſwer. According to *Euclid*, a Plain can touch a Sphere only in one Point, call'd the Contact; and that Perſon only who ſtands to that Point (with reſpect to the Center of that Sphere) can ſtand upright, and whereas the ſenſible Horizon changes, as oft as we change our Place, becauſe of the Convexity of the Earth's Periphery: and ſuppoſing each Man to ſtand, as perpendicular as a Plumb-line to his own *Horizon*; and ſeeing 'tis an undoubted Axiom, that *All heavy Things tend towards the Center of the Earth*, where all perpendicular Lines, if extended, would meet: Theſe Things conſider'd, I ſay, 'tis abſolutely impoſſible for two Men to ſtand perpendicular, to the ſame Plain, without contradicting the Axioms aforeſaid; for could they ſtand perpendicular, then would they be parallel to each other; and were Parallels extended to the Center of the Earth, they would never meet, as all Plumb-lines ſo extended, would.

See Euclid, *by* W. Whiſton, *Lib.* 1. 19. *and* 3. 2, 16, 37. *in their Conſectaries.*

'Tis true, this intellectual Truth is eaſily demonſtrated *to the Mind*, tho' not ſo eaſy to be mechanically prov'd to *the Eye*; becauſe the Height of a Man bears no *ſenſible* Proportion to the Earth's Semidiameter. This is the longer inſiſted on for the Sake of the following *Paradox*.

34. *There is a certain* European *City, whoſe Buildings being generally of firm Stone, are (for the moſt part) of a prodigious Height, and exceeding ſtrong; and yet 'tis moſt certain, that the Walls of thoſe Buildings are not parallel to one another, nor perpendicular to the Plain on which they are built.*

Anſwer. All Walls are endeavoured to be built perpendicular to the Tangent (and point to the Center) of the Earth; where they, if continued, would meet in a Point; but if extended to the Moon and Stars, would grow wider and wider aſunder, the nearer they approach'd them; and conſequently are not Parallels, which, if infinitely extended, would never meet, nor part further aſunder, but keep ſtill the ſame Equidiſtance: Alſo in one Point only can a Perpendicular, to the Earth, be raiſed on a horizontal Plain, as appears by the laſt *Paradox.* But to be more particular; 'Tis not improbable our ingenious Author might, in this *Paradox*, intend the City of *Edinburgh*, noted for ſtrong, high, and Stone Buildings; ſome being, as is reported, fourteen Stories high, built on an Hill; and therefore the Walls are not perpendicular to the Plain of the Hill, but to the Baſe of it; and the Walls are not parallel to each other, for the Reaſons aforeſaid.

35. *There is a certain City, in the* Southern *Part of* China, *whoſe Inhabitants (both Male and Female) do obſerve almoſt the ſame Poſture and Gate in Walking as we* Europeans; *and yet they frequently appear to Strangers as if they walk'd on their Heads.*

Anſwer. In *China* (or any other Places where the Inhabitants ſtand near the Sea) Strangers looking in it muſt ſee them as tho' their Heads were downward, by the refracted Viſion; Or *China* being ſituate almoſt in oppoſite Meridians to us; and therefore, to Strangers in Geography, to tell them there are People walking with their Feet towards ours, they will aſk (according to the Appearance of the Thing to them) Do they then walk on their Heads? for our own are uppermoſt, and their Heads muſt be under our Feet: Then whereas the *Globe* being round all our Heads are next Heaven, and Feet next the Earth, and no upper nor under on the *Globe*, any more than on a Wheel in Motion.

36. *There are ten Places of the Earth diſtant from one another three Hundred Miles and upwards, and yet none of them hath either Longitude or Latitude.*

Anſwer. The Places are ſaid to be *of the Earth*, not *upon the Earth*; for Longitude and Latitude are reckon'd on the Surface of the *Globe*

Globe only; ſo the *Axis* of the Earth, or any other imaginary Line, being above ſeven thouſand *Italian* Miles, will not only anſwer this *Paradox*, but inſtead of ten, if he had ſaid twenty, it would have kept within the Poſſibility of the Demand.

37. *There are two diſtinct Places of the Earth, lying under the ſame Meridian, whoſe Difference of Latitude is ſixty Degrees completely; and yet the true Diſtance, betwixt thoſe two Places, doth not really ſurpaſs* 60 Italian Miles.

Anſwer. The *two Places* are not meant on the Superfices of the Earth, as you may perceive by the Word *of* (and not *upon*) ſo the Places will be ſo near the Center of the Earth, as two Lines ſuppoſed to come one from no Degree of Latitude, the other from ſixty Degrees of Latitude, and to meet in the Center, may approach within the Diſtance of ſixty *Italian* Miles. Or if the Places muſt be on the Globe of the Earth, we muſt diſtinguiſh between Latitude when apply'd to a Country, and when apply'd to a City; the laſt, is the Diſtance of that City from the Equator, *North* or *South*, the firſt is only the Breadth of a Country from *Eaſt* to *Weſt*: Thus, the two Countries may be *Italy* and *Germany*, which lye under the ſame Degree of Longitude; the Breadth, or Latitude of *Germany*, is ſaid to be five hundred and ten Miles; and the

Breadth, or Latitude of *Italy*, is ſaid to be an hundred and thirty four Miles; their Difference of Breadth or Latitude, is three hundred and ſeventy ſix Miles, or above ſixty Degrees; and yet *Italy* is not ſixty Miles diſtant from *Germany*, for they are parted but by the *Alps*.

38. *There are alſo two diſtinct Places of the Earth, lying under the Equinoctial Line, whoſe Difference of Longitude is completely eigty-ſix Degrees; and yet the true Diſtance, between theſe two Places is not full* 86 Italian *Miles.*

Anſwer. As in the laſt, ſo in this, the Longitude of a City is its Diſtance *Eaſt* or *Weſt* from the firſt Meridian; but the Longitude, or Length of a Country, is its Diſtance from *North* to *South*; Thus, ſuppoſe the Iſle of St. *Thomas*, whoſe Longitude, or Length is not above a Degree and an half, and the Country of *Ethiopia Exterior*, which is above 90 Degrees long, reckoning its Length down from *Nubia* to the *Cape of Good-Hope*, and up all along the Coaſt of *Zanguebar*; wherefore the Difference of Longitude, or Length may be ſaid to be completely, 86 Degrees and a half, and yet the true Diſtance of the ſaid Iſland from the ſaid Country, is not much above a Degree: Or the Places are not ſuppoſed on the Surface of the Earth, but nearer to the Center, where the Longitudes coincide.

39. *There are three diſtinct Places of the Earth, all differ both in Longitude and Latitude, and diſtant from one another two thouſand Miles completely, and yet they all bear upon one and the ſame Point of the* Compaſs.

Anſwer. All Places, tho' they differ both in Longitude and Latitude, at what Diſtance ſoever with reſpect to either Poles, bear upon the ſame Point of the Compaſs: Or they may be in the ſame Spiral Rumb, elſe underſtood as *in the Earth*, and not upon *it*. For to an Eye, ſituate under the $\left\{\begin{array}{l}\textit{North}\\\textit{South}\end{array}\right\}$ *Pole*, all Places, howſoever ſituated, will bear on the $\left\{\begin{array}{l}\textit{South}\\\textit{North}\end{array}\right\}$ Point of the *Compaſs*, becauſe every Azimuth is a Meridian, and the *Pole* and *Zenith* coincide, as aforeſaid.

40. *There are three diſtinct Places on the Continent of* Europe, *equidiſtant from one another (they making a true equilateral Triangle, each of whoſe Sides doth conſiſt of a thouſand Miles) and yet there is a fourth Place ſo ſituated in reſpect of the other three, that a Man may travel, on Foot, from it to any of the other three in the Space of one artificial Day, at a certain Time of the Year, and that without the leaſt Hurry or Fatigue whatſoever.*

Anſwer. By an artificial Day, is meant from Sun riſing to Sun ſetting: Now beyond the Polar Circles, and nearer to the Poles, the Days are increaſed from twenty-four Hours to ſix Months, without Sun ſetting under the Poles, in which Time, one may travel above 4000 Miles and travel only a Mile an Hour one with another.

Or ſuppoſe the three Places to be in *Sweden*, *Norway*, and *Muſcovy*, where their Day is about two Months long near the Summer Solſtice, let the fourth Place be equi-diſtant from the other three: Now if the Sides of an equilateral Triangle be one thouſand, the Radius of its circumſcribing Circle will be 577 Miles; and if a Man travels but ten Miles a Day, he may readily travel from the Center of the Triangle to any of thoſe three Places in one ſuch Day.

41. *There are three diſtinct Places on the Continent of* Europe, *lying under the ſame Meridian, and at ſuch a Diſtance, that the Latitude of the third ſurpaſſeth that of the ſecond, by ſo many Degrees and Minutes, exactly as the ſecond ſurpaſſeth the firſt; and yet the true Diſtance of the firſt and third, from the ſecond or intermediate Place is not the ſame by a great many Miles.*

Anſwer. The oblate, ſpherodical Figure of the Earth may cauſe ſuch a Difference. Or ſuppoſe *London*, *Paris*, and *Bourbon*, all under the ſame brazen Meridian, equally

Different in Latitude; yet the Diſtance of *London* from *Paris*, will exceed the Diſtance of *Bourbon* from *Paris*, by near an hundred Miles; becauſe *London* is about two Degrees *Weſterly* of *Paris*, about the Breadth of the Brazen Meridian; whereas *Bourbon* and *Paris* are in the ſame Longitude, and conſequently nearer, by almoſt two Degrees.

42. *There are two diſtinct Places on the* Continent *of* Europe, *ſo ſituated, in reſpect of one another, that tho' the firſt doth lye* Eaſt *from the ſecond, yet the ſecond is not* Weſt *from the firſt.*

Anſwer. If any two Places be in the ſame Parallel of Latitude, reſpecting the Rumb, the firſt may and muſt bear off the ſecond *Eaſt* and *Weſt*; and yet the ſecond, reſpecting the Angle of Poſition, or the bearing of one Place from the *Zenith* of the other, on the *Globe*, may be far ſhort of being due *Weſt*, as *Liſbon* in *Portugal*, and *Smyrna* in *Natolia*, are in the ſame Parallel of Latitude, *i. e.* thirty nine Degrees, and therefore, by the Rumb, they bear *Eaſt* and *Weſt*. But on the *Globe*, *Smyrna* bears off the *Zenith* of *Liſbon* ſeventy five Degrees *North Eaſt*, and *Liſbon* bears off the *Zenith* of *Smyrna* eighty Degrees *South Weſt*, which is evident from the Globe and Circle ſailing.

Note, A Rumb Line makes equal Angles

with all Meridians on the *Globe*; and an equal Part thereof altereth the Latitude equally: But the Circle of Poſition makes unequal Angles, *i. e.* greater Angles with all other Meridians than with that from which it was drawn.

43. *There is a certain* European *Iſland, the Northermoſt Part thereof, doth frequently alter both its Longitude and Latitude.*

Anſwer. By *European* Iſland, may be meant either the Map of it, as *Iceland*, whoſe *North* Part thereof, uſed to be ſet above the Artic Circle, but now is below it, and nearer the *Eaſt*. Or the Iſland it ſelf; then it muſt mean ſome floating Iſland, as *Boetbius* tells us, there is one in *Lomond*, the biggeſt Lake in *Scotland*; which alſo our ingenious Author mentions, as one of the Rarities of *Scotland*; in which Lake are alſo Fiſhes without Fins, and is frequently tempeſtuous in a Calm: Or any Iſland, whoſe *North Eaſterly* Part is overflow'd by the Sea; this will alter both its Latitude and Longitude.

44. *There is a certain Place in the Iſland of* Great-Britain, *where the Stars are always viſible, at any Time of the Day, if the* Horizon *be not over-caſt with Clouds.*

Anſwer. That Place may be ſome Coal-pit, Well, deep Cave, or high Chimney, or

Dr. *Halley*'s *Royal Obſervatory*; ſuch as *Tycho* had at *Denmark*; which was a deep Well or Dungeon, beſet with Looking-Glaſſes, where he ſat and obſerv'd the Stars, in all Seaſons.

45. *It may be clearly demonſtrated, by the* Torreſtrial Globe, *that it is not above twenty four Hours Sailing, from the River* Thames *in* England, *to the City of* Meſſina, *in* Sicily, *at a certain Time of the Year; provided there be a briſk North Wind, a light Frigate, and an Azimuth Compaſs.*

Anſwer. Nor twenty four Hours neither, if the Voyage be perform'd on the *Globe*, and the Time meaſur'd by its Hour Index; Or if it mean the Tides ſailing betwixt thoſe two Places: Or becauſe one may paſs from *France* to *Italy* by Land, and the Sailing betwixt *Britain* and *France*, alſo *Italy* and *Sicily*, is not above twenty four Hours: Or if by twenty four Hours, be meant the natural Day under the *Poles*, which is a whole Year, this Voyage may be perform'd more than once in that Time: Or, Laſtly, the Difference betwixt the *Julian* and *Gregorian* Accounts being eleven Days, whereunto add another, which makes twelve; a Time ſufficient for ſuch a Voyage; provided you have ſtill, a good Wind, light Frigate, and proper Inſtruments. This may eaſily be demonſtrated by the Terreſtrial *Globe*, on whoſe *Horizon* both Accounts are laid down.

' Theſe (adds our Author) are the chief ' Paradoxical Poſitions, in Matters of Geo- ' graphy, which mainly depend on a thorough ' Knowledge of the *Globe*; and though it ' be highly probable that they'll appear to ' ſome, as the greateſt of Fables, yet we ' may boldly affirm, that they are not only ' equally certain, with the (aforeſaid) The- ' orems, but alſo we are well aſſured, that ' there's no mathematical Demonſtration of ' *Euclid* more infallibly true in itſelf, than ' is every one of them.

Howbeit, leſt ſome of the foregoing Solutions ſhould not anſwer the Dignity of this Encomium or Character; hear what the ſame ingenious Author, who doubtleſs knew his own Senſe beſt, acknowledges in his Preface.

' If therefore theſe Paradoxes, above- ' mention'd, ſhall obtain the End propos'd ' (the rouſing of the *Mind* to *Think*) it mat- ' ters the leſs, if ſome of them, upon ſtrict ' Enquiry, ſhould be found to conſiſt of ' equivocal Terms, or, perhaps, prove little ' more than a Quibble at the Bottom." To which I ſubjoyn, if any of the preceding Anſwers ſeem not ſufficient, or ſatisfactory, I deſire our Author's End may be ſtill purſued:

ſued: May it rouſe the Mind of my judicious Reader, to *Think*, and offer a better; according to the Saying of the Poet, with which I conclude this firſt Part.

——Si quid noviſti rectius iſtis,
Candidus imperti; Si non, his utere mecum.

PART. II.

NEW PARADOXES SOLVED.

46. IN what Degree of Latitude
Does that ſoft Female dwell,
Who, upon certain Days, might view,
If ſhe obſerveth well,
The Morning Sun o'th' ſelf ſame Point
Of Compaſs twice to be,
Likewiſe his Evening Azimuth
Twice in the ſame Degree.

Tell me ſweet *Engliſh* Ladies, for you are
Than her more charming far, and far more fair.
Anſwer. *Between the* Tropics. See Paradox 12.

47. In the *Atlantick* Ocean, from our Shore
Diſtant five hundred Leagues, or little more,
Lie the *Canary* Iſles, bleſt with good Air,
Sweet whiſtling Birds, rich Wines beyond compare.
A Mountain, the World's Wonder, ſituate there,
One of the higheſt in this earthly Sphere;
By us 'tis call'd the *Teneriffa*'s *Pike*,
Whoſe lofty Head i'th' Atmoſphere ſo high,
That it ſurmounts all groſſer Clouds o'th' Sky;
Three Degees and an half you may it ken,
Or (what's the ſame) two hundred Miles and ten;
For the Refraction, Angle you may make,
Allowance (what is thought but meet to take)
Thirty five Minutes, juſt of a Degree:
Now I demand what Height the Pike muſt be.
Anſwer. 4. 28. *Miles high.*

48. I have twelve Times ſeen *Biſſextile*, pray tell how that can be,
Since twelve times four make forty eight, and I'am but forty three?

Anſwer. *If a Perſon be born on the 25th of* February, *and travel* Weſtwards *the* Globe *about, he may ſee twelve* Biſſextile *Years before he be completely forty four Years of Age, if he was born in a* Biſſextile *Year.*

49. A Golden Ball ſhall weigh two Pounds in *Troy*,
Where from th' Æquator ten Degrees does lye;
But at *London* a different Weight is found,
Of that ſame Ball, which Difference pray expound.

Anſwer. *The Earth is not exactly round; but its Diameter at the Equinoctial Circle, is, to its Diameter through the* Poles, *as ſix hundred ninety two to ſix hundred eighty nine; therefore, according to the incomparable Sir* Iſaac Newton, *the Proportion of Gravity, betwixt the* Æquator *and* Poles, *is as five hundred to five hundred and one, and the Increaſe, near as the Square of the right Sine of the Latitude; and that the Ball would weigh more at* London *than at ten Degrees Latitude, by thirteen Grains.*

This then is (I preſume) occaſion'd from the Earth's oblate Figure, and the Diminution of the centri-petal Force or Gravity, which is univerſal. 1. *In every ſingle Particle of Matter.* 2. *As to its Extent.* 3. *As to all kind of Bodies.* 4. *As to Time.* 5. *As to Quantity of Matter.* 6. *In a duplicate Proportion to their Diſtances.*

50. Dic quibus hoc Animal Terræ naſcatur in Oris;
Maſculus eſt mater cui Mulierque pater.

Anſwer. *Progenitos vidi numeroſos vere Gyrinos*
Hæc rana, hic Buſo, Mater hic, illa Pater.

A Tadpole, *young* Frog, *or* Toad.

51. *Leander* to his *Hero* writ, and ſhe
As oft writ back, to ſhew her Conſtancy;
When with them both the Poſt three Times had been,
They had no more than each a Letter ſeen.

Anſwer. *The Paradox means with* both, *not* each *three Times: The Poſt firſt took* Leander's *Letter, ſecond* Hero's *Anſwer, and third brought it to* Leander.

52. The Day that I was born, my Father he
Laid by five Pounds, and ſaid it was for me;
And whene'er my Birth-Day came he ne'er fail'd
To add five Pounds, (his Love ſo much prevail'd
At twenty-four, and upon my Birth-Day,
I wedded was, my Portion he would pay,

Juſt thirty-five Pounds (I full twenty four)
How comes it then my Portion was no more.

Anſwer. *The Perſon was born in* Leap Year, *on the 29th of* February.

53. A certain Mount in *Devonſhire* doth ſtand,
Whoſe lofty Head o'erlooks the neighb'ring Land?
And ſuch is the known Property o'th' Hill,
That if a Veſſel you with Liquor fill,
At the Hill's Top, or Vertex of the Place,
It holds leſs than if filled at the Baſe:
Now what's the Cauſe of this Deficiency,
Pray let me know in the next Diary.

Anſwer. *Water has its Surface always of a ſpherical Figure, and that fartheſt from the Earth's Center is leaſt Swelling, and conſequently holds leaſt above the Brim of the Veſſel: The Top Diameter of the Veſſel being the Chord of a greater Circle, on the Top of an high Hill than at the Bottom, which is a ſomewhat leſs Circle, and conſequently more Convex, will therefore hold more above the Brim, at the Baſe, than at the Top of the Mountain.*

54. When Tyrant *Noll*, at fatal *Worceſter* Fight,
His Crew harangu'd, againſt both Law and Right,
His quaint numerique Voice no two did hear,
Tho' ſeveral thouſand Miſcreants were there;
For every Man, and every ſeveral Ear,
Did not the ſame but diſtant Voices hear.

Anſwer. *Sound being afflated in Concentric Circles as a Stone falling into Water, moves it all around; ſo that Part of the Circle, which touch'd one Man's Ear could not touch another's, who could not be in the ſame individual Spot of Ground.*

55. By what Means may I know moſt certainly,
That two Men in the World there are,
Whoſe Heads of Hair (if numbered they be)
Are equal to a ſingle Hair;
Altho' ſome Millions on each Head there were.

Anſwer. *If the Number in the World, do exceed*
The Number of Hairs that are on a Man's Head,
You may ſafely affirm then, that two Men there are
That have the ſame Number, to a ſingle Hair.

To explain and prove the Truth of this Hypothetical Propoſition; That if there be more Perſons in the World than the Numbers of Hairs on the Head of any one alive, *then muſt there be two Men in the World, the Number of Hairs of one of whoſe Heads, is exactly equal (to a Hair) with thoſe on the other's Head.*

Suppoſe a very ſmall Number of Men, for Inſtance eight, and the Hairs on none of their Heads exceed ſix; the ſame Conſequence will follow by the ſame Ratiocination, as if you made Uſe of a greater.

Next, Suppoſe one of the eight has but one Hair, if any other of the ſeven have only one Hair; we have already found out what we deſir'd; if not, ſet this Man by, and out of the ſeven remaining, chuſe another, which we ſuppoſe has two Hairs; now if any one of the other ſix hath two Hairs, and no more, we have what we ſought for; but if not, ſet this Man aſide, and take another, out of the ſix remaining Men, which we ſuppoſe has three Hairs; if then any of the other five has got three Hairs, as well as he, our Enquiry is at an End; if not, ſet him aſide, and chuſe another out of the five, which we ſuppoſe has four Hairs; if any of the other four ſhould chance to have four Hairs, then have we two Men that have each an equal Number, the Thing ſought; if not, then they all four muſt have either five or ſix Hairs; for beyond ſix no Man can have: Then put this Man by, and take a fifth, whom we ſuppoſe to have five Hairs; now if any of the other three have five, we have our Deſire; if not, then it follows, that the three remaining Men, muſt of Neceſſity, have

have each Man ſix Hairs; for before we have proved that they had neither one, nor two, nor three, nor four, nor five, and therefore they muſt have ſix, beyond which they cannot exceed.

Now, if inſtead of ſuppoſing the firſt Man had one Hair, you had ſuppoſed he had ſix, five or four, &c. *you would, by the ſame Way of Arguing, come at laſt to the ſame Concluſion.*

56. *Phillis*, the ſhy and ſcornful fair,
Was by brave *Strephon* held moſt dear,
But ſtill, as he preſt on his Joy,
She grew more malepert and coy:
And when he talk'd of Marriage to her,
She got more diſtant from her Wooer.
But what he ſu'd for, thus, in vain,
He did by Accident obtain;
For walking with his *Phillis* dear,
One Afternoon, to take the Air,
In a vaſt Champain Country, where
Nor Hedge, nor Houſe, nor Tree were near,
When ſtrait dark gloomy Clouds appear,
Which did preſage a Storm was near.
Anon (no Shelter being nigh)
It rain'd and blow'd impetuouſly:
Which made the ſhy Dame him invoke,
To ſhelter her with his long Cloke.
Soon after this the Storm grew higher,
Which made the Nymph cling ſtill the nigher:
Then roar'd around them Peals of Thunder,
As if the Earth had burſt aſunder.
This made the trembling Maid embrace,
And in his Boſom ſhrowd her Face.
The Swain, tho' almoſt ſtun'd with Fear,
Being thus warmly claſp'd by's Dear,

Took Courage, and to her thus ſpoke,
Phillis, I all the Gods invoke,
To Witneſs that my Paſſion's true,
And would forego the World for you;
No Flattery I need pretend,
For ſurely now the World will end.
Oh *Strephon!* (cries the trembling Fair)
My Thoughts I'll tell you, without Fear;
I Love as much, as dear as you,
But Pride forbids what Love did ſue:
And if this dreadful Storm blow o'er,
I ne'er will perſecute you more.
Then did they intermingle Kiſſes,
A ſure Preſage of future Bliſſes.
Straightway the Heav'ns began to clear,
And Joy ſucceeded now their Fear.
This was, as I do well remember,
About the fifth Day of *December.*
They then agreed to be in Wedlock joyn'd,
The longeſt Day he in the Year could find;
And in the following Week the Marriage was,
Pray tell me now how this could'come to paſs.

Anſwer. *From Noon to Noon,* Aſtronomers *do ſay,*
Are not juſt twenty-four Hours, ev'ry Day,
But longer ſome, ſome ſooner haſte away.
Good Pendulums, *as I do well remember,*
About th' eleventh or twelfth Days of December,
The Space of thirty Seconds, meaſure more
In Number, than they did the Day before;
But no Days elſe, which plainly make appear
That theſe the longeſt are in all the Year.
Strephon *rejoyc'd, that he ſo ſoon had found*
The Day on which his nuptial Joys were crown'd.
Fair Phillis, *too, tranſported with Delight,*
No leſs rejoyc'd it prov'd the longeſt Night.

Some Reaſons follow, ſerving to explain the foregoing Solution.

1. *If the Sun or Earth moved along the Equinoctial, by a conſtant, uniform, equal Motion; yet the Time, from Noon to Noon, would not be exactly twenty-four Hours, but twenty-four Hours four Minutes; becauſe in one Revolution the Sun would advance about one Degree further than the Day before.*

2. *If the Sun be ſuppoſed to move equally along the Ecliptic; yet, becauſe of its Obliquity, equal Arches of the Ecliptic would not anſwer to equal Arches of the Equinoctial.*

3. *The Sun doth not move with a conſtant, uniform, equal Motion, in the Ecliptic, but ſometimes faſter and ſometimes ſlower; becauſe it moves in an Ellipſis; and in going one Half of it, ſpends eight Days more in the* Summer *than in the* Winter *Half Year: Therefore, it unavoidably follows, that the Days are unequal, and not exactly twenty-four Hours.*

57. A Landed Man two Daughters had,
And both were very fair,
He gave to each a Piece of Land,
One round, the other ſquare:
At twenty Shillings an Acre, juſt,
Each Piece its Value had,
The Shillings which did compaſs each,
For it exactly paid.
If croſs a Shilling be an Inch,
(As it is very near)
Which was the better Fortune, ſhe
That had the Round or Square;

	l.	*s.*	*d.*
Anſw. *The Value of the ſquare Piece is*	250905	12	0
round	197060	12	$7\frac{1}{2}$
Difference	53844	19	$4\frac{1}{2}$

The *Algebraic* Proceſs is,

The Perches Engliſh *Meaſure.*

In the Square.

a=*Nr. of Shil. that purchas'd it*

a=*Nr. of Inch. about the 4 ſides*

n=*N. In. in ſq. acre*=6272640

f=*Number of Shillings that purchas'd an Acre.*

Then $\frac{a}{4}$ =*one ſide which ſquar'd gives* $\frac{aa}{16}$ *which are the ſquare Inches in the Cloſe, this divided by n gives* $\frac{aa}{16n}$ *the Nr. of Acres, which multiplied by f gives* $\frac{aaf}{16n} = a$ *the whole value of the ſquare Piece in Shillings, and being reduced gives*

$$a = \frac{16n}{f} = 250905\ 12\ 0$$

In the round Piece.

a=*Circumference in Inches.*

a=*Nr. Shil. that purchas'd it.*

d=*Diameter of the Circle*= 1

c=*Circ. of ſaid Dia.* 3. 14159

Then ſay

As c to 1 : : *a* : $\frac{a}{c} = d$

then ½ *Circumference into* ½ *the Diameter gives the Area*

$$\frac{a}{2} \times \frac{a}{2c} = \frac{aa}{4c}\ \textit{Area.}$$

This Area divided by n, and multiplied by f, as in the form r, gives the Purchaſe in Shillings, viz.

$$\frac{aaf}{4cn} = a\ \textit{which reduced is} = \frac{4cn}{f} = 197060\ 12\ 7\tfrac{1}{2}$$

58. Suppoſe a round Ball for to move in the Air,
In a certain Proportion, which I ſhall declare,
Let the firſt Hour be twelve Miles, the next to move ten,
And ſo in Proportion, from whence it began,
As twelve is to ten. Now try if you can
Tell the Miles it will move, ſuppoſe it to be
Continu'd in Motion to Eternity.

Anſwer. *The Ball that is moving* Eternally *ſo*
In Proportion of twelve to ten juſtly will go.
Threeſcore and twelve Miles, and no more I declare, &c.

The Reaſon of this Paradox is, that in infinite Geometrical Progreſſion, if the Proportion do continually decreaſe, the laſt or leaſt Term, will be infinitely ſmall, or, in Effect, nothing, and ſo produce a finite Number.

$$\textit{Suppoſe}\ \begin{array}{l} A=\textit{firſt or greateſt}=\ \ 2\\ B=\textit{ſecond Term}\ \ \ =10\end{array}\ \textit{it will be}\ \frac{AA}{A-B}=\textit{Sum of all}$$

the Terms=72.

59. If, in one Second's Time, an Iron Ball
Did five and twenty *Engliſh* half Feet-fall:
How long muſt be the Time it then muſt take
A Journey of ten thouſand Miles to make?

Anſwer. *I think the Time in which the Iron Ball,*
If gently dropt, ten thouſand Miles will fall,
Muſt be allow'd in Minutes thirty-four,
And Seconds fifteen neither leſs nor more.

This depends on the following Propoſition, that the Acceleration of the Deſcent of heavy Bodies, are as the Square of the Times; *the Reaſon is, the Action of Gravity being continued in every Space of Time, the falling Body receives a new Impulſe equal to what it had before, in the ſame Space of Time, received from the firſt Power.*

Thus, Suppoſe in the firſt Second of Time a Body hath *acquired a Velocity, which, in that time, would carry it thirty two Foot (and were there no new Force, it would continue to deſcend, at that Rate, with an equable Motion) but, in the next Second of Time, the ſame Power of Gravity, continually acting thereon, ſuperadds a new Velocity equal to the former; ſo that at the End of the two Seconds, the Velocity is double, to what it was at the End of the firſt.*

Hence, then the Queſtion is eaſily ſolv'd; for turn the Miles into Feet 52800000 *and divide it by* 12..5, *the Quotient is the Square of the Seconds the Body will be falling* (viz. 4224000) *the Square the Root whereof gives Seconds* (2059) *and dividing by* 60, *gives Minutes, namely* 34: 19 *Seconds a little above half an Hour.*

60. Before my Father was begot,
I'm ſure I was begotten.
And born before my Mother,
They both are dead and rotten:
And I am lying in that Bed
Where I got my Grandmother's Maidenhead,

Anſwer. Adam, *I know, was ne'er begot,*
And Eve *of him was made,*
And ſince by him they brought in Death:
They both, long ſince, are dead,
'Tis certain, Abel *was begot*
Of thoſe his two ſaid Parents,
He dy'd by Force, and was the firſt
That yielded to Death's Warrants:
Therefore he got the Maidenhead
Of's Grandmother, the Earth,
Being laid therein the firſt that was,
Tho' not the firſt by Birth.

61. I. As I was muſing all alone,
What Paradox to pitch upon,
To entertain the Fair,
Within my Study flies a Bee,
Whoſe Wings made pretty Melody,
As ever I did hear.
II. Pleas'd with the Noiſe, I drew more nigh,
When inſtantly came in a Fly,
That charm'd me with her Sound;
The then united Tones produc'd
A perfect Fifth, as e'er was us'd,
Or could by Art be found.
III. The nimble Fly's Wings quicker were
Than thoſe of her Competitor,
As may by this appear;
For an acuter Tone they made,
And in a ſharper Key they play'd,
(Which made the Matter clear.)
IV. If in a Minute's Time thought I,
The nimble Wings of the ſame Fly
Ten thouſand Beats did go:
What Number then muſt be the Swings
In two Hour's Space, of the Bee's Wings?
Is what I fain would know.

Anſwer. *Believe me, Sir, the little Bee,*
With her harmonious Wings,
Did make exact, in two Hours Space,
Eight Hundred thouſand Swings.

The Foundation of this depends upon the Diviſion of a Monochord, &c. *Divide the String of a Viol into three equal Parts, and then ſtrike the whole String (and obſerve the Sound) ſtop the String again, in one of the third Parts, and ſtrike the Length that hath two Parts of the Whole open; it will ſound a perfect Fifth to the former*

Tone; ſo that the Proportion is as three to two; ſo is the Diapaſon to the Diapante, or the Muſicians Eighth to their Fifth: So that the Paradox is eaſily anſwer'd; for as three to two, ſo is the Number of Vibrations that the Fly makes, in two Hours, to the Number which the Bee makes in the ſame Time, viz. 800000.

62. I once eſſay'd (but 'twas in vain)
 To win the Prize propos'd,
However I'll venture, once again,
 'Cauſe merrily diſpos'd.
The Mark whereat I now do aim,
 The promiſed Reward,
Eggs me to play a forced Game,
 And draw my only Card;
Aſtoniſhing it ſure muſt be
 As Paradox e'er was,
And thro' Surprize you ſcarce can ſee
 How it ſhould come to paſs;
That I, the only Offspring, then,
 Was marry'd to my Mother,
And yet ſhe never marry'd Man,
 Nor ever took another.
Nay, ſtill a ſtranger Thing you'll find,
 That when I wedded was,
I then was of the Female kind,
 My Mother was the Mas;
Yet the moſt fruitful Wife never had
 So many Sons as I;
'Tis hard methinks, to be conceiv'd,
 Tho' ſpoke without a Lye.
But know, the Cream o'th' Jeſt is here,
 No Body was my Father;
Then wonder, yet, how't may appear
 That ever I had Mother.

How could it ever happen ſo
I'll have you all to gueſs,
And yet, perhaps, you can't find who
My loving Huſband was.
Here, Ladies, you may ruminate,
And ſtill remain in Doubt,
Which if you can't inveſtigate,
I'll dare to make all out.

Anſwer. *Your Paradox muſt needs ſurprize*
And much delight the Fair,
I think you merit well the Prize,
For 'tis beyond Compare,
Aſtoniſh'd I found out at laſt
Your meaning muſt be Eve,
Being taken out of Adams *Waſte*
Did Female Shape receive,
The only Off-ſpring then ſhe was
And Adam *was her Mother*
She Woman was, and he the Mas,
Yet was he not her Father,
A fruitful Wife ſhe was, 'tis plain
Unto her Huſband Adam,
Her eldeſt Son we read was Cain
God knows a very bad one,
The Truth I hope, I have obtain'd
If not, I beg your Pardon.

63. Walking the other Day, to take the Air,
(Bright ſhone the Sun, the Weather very fair)
At Diſtance I a diſmal Cloud did ſpy,
Which (as methought) againſt the Wind did fly.

While I upon my Watch did look, to ſee
How Time did paſs away, lo, inſtantly
A dreadful Flaſh of Lightening pierc'd the Cloud,
Juſt fourteen Seconds, after which, aloud
The Thunder roar'd, now I inform'd would be,
How many Feet the Cloud did burſt from me.

Anſwer. *When Heat, and Moiſture, are in one conjoyn'd,*
And ſable Mountains chac'd before the Wind,
When two contraries were together pent,
Their inbred Jarrs, ſoon ſhew'd their Diſcontent.
Then Mighty Jove, *to ſcourge their Mutiny,*
Bombards their Fort with his Artillery;
For when by Force th' embodied Cloud is rent,
A Peal of Thunder ſhakes the Firmament.
See here the Diſtance when that Rupture brake
When angry JOVE *thus to his Creatures ſpake,*
Huſh, ceaſe this Rage. *They ſtrait are calm and ſtill,*
They all obey, and wait upon his Will.

According to Sir Iſaac Newton, *Sound flies* 968 *Feet in a Second*; *wherefore* 968 × 14 = 13552 *Feet, or Miles* 7, 6 *Furlongs* 160 *Feet.*

Perhaps it may not be unwelcom to the Reader to deliver ſome Obſervations concerning *Sound,* and the *Progreſſion* of it.

Sound is produc'd by the little Vibrations, which the Parts of the ſonorous Body occaſions in the Air; and the *Strength* or *Weakneſs*; in Proportion to the Quantity of the Air ſtruck, and the Strength of the *Vibrations.*

Mr. *William Derham,* F.R.S. obſerves there is a conſiderable Difference in the Accounts given by good Authors about the Velocity of the Motion of *Sound* in a *Second* of Time, as followeth.

Sir *Iſaac Newton*'s firſt Edition of Mathematical Principles, gives 968 Feet in a Second of Time.

His ſecond Edition corrected ———— 1142

Francis Roberts ———— 1300

Mr. *Bailie*	1200
Dr. *Walker*	1331
Mr. *Flamſtead* Aſtronomer Royal	1142
Edm. Halley	
Mr. *Derham*, F.R.S.	
Florentine Academy	1185
Royal Academy at *Paris*	

This Diverſity proceeds, probably, from not uſing good Pendulums, but *Strings* and *Plummets*, 2*d.* From diſregarding the Winds, 3*d.* Diſtance betwixt *Sound* and *Obſervation* too ſhort. And the Difference betwixt the three laſt being ſo ſmall, might ariſe from uſing Pendulums equally good, and Experiments equally large.

The ſame worthy Author, by manifold Experiments, and repeated Obſervations, found no Difference whether the *Gun* was fir'd towards or againſt him, in the Velocity of the *Sound*, nor in any different Elevation, nor in different Quantity, or Strength of Powder (though it did in Strength of Noiſe) neither betwixt Day or Night, Clear or Cloudy, Winter or Summer, Rain or Snow: All kinds of *Sounds* came in the ſame Time to his Ear, from the ſame Diſtance, whether tenſe or ſtrong, languid or weak.

By very accurate Obſervations, he found the Motion of *Sound*, to be equable in all Reſpects; that is, an *Engliſh* Mile in nine half *Seconds*, two Miles in 18, three Miles in 27, *&c.* And he is confirm'd in the Belief, that *Sound* moves the neareſt Way, and its Velocity equal in Acclivities or Declivities, and that Difference of Climates or Weather, makes no Difference in the Velocity, though thick, cloudy or ſnowy Weather did dull the Noiſe or Sound.

But I know not why Winds might not ſo affect *Sound*, that is, by conſpiring with its Motion, advance it to 1200, or retard it by its being contrary, that it may not move above 1120 in a Second of Time.

The Uſe of all this may be to meaſure the Diſtance of Thunder, Forts, Batteries, or Ships at Sea, *&c.*

Any *Space* or *Diſtance*, propoſed to find how long *Sound* will be flying that Diſtance, ſay.

As 1142: is to 1::ſo is the Diſtance in Feet to the Time in Seconds.

Any Time propoſed to find the Space or Diſtance that a Sound will fly in that Time, ſay,

As 1: is to 1142:: ſo is the given Time in *Seconds* to the Diſtance in Feet.

From what has been ſaid, we may conclude, that *Sound* moves 1142 Feet in one *Second,* eight Miles in 37 *Seconds,* 13 Miles in one minute *Fere,* and near 780 Miles in an Hour.

And if we number ten *Seconds* between the Fire of the Cannon ſeen and the *Second* heard, it is manifeſt that the Gun is 11420 Feet diſtant, or ſomewhat more than two Miles.

As likewiſe, if 5 *Seconds* paſs betwixt one ſeeing the Flaſh of Lightning, and hearing the Thunder, we may reckon the Thunder-Cloud is about 5710 Feet, or a little above one Mile diſtant from us.

64. Enflam'd in Love and wrapt in Extacy,
Encircled in *Eliza*'s Arms I lay;
When envious Chance did our ſad parting cauſe,
Againſt both Nature's and Affection's Laws;
But our next Meeting, we did then aſſign
Seventy eight hundred Beats and twenty-nine
Of a nice Pendulum, whoſe Length I ſee
Twenty five Inches three Eighths exact to be.
Aſſiſt me quickly, oh you Sons of Art,
You who have felt the Sting of *Cupid*'s Dart,
You who to Tenderneſs and Pity move,
And know the Pangs of diſappointed Love;
For if, too ſoon or late, I chance to go,
Deſpair and Ruin certainly enſue:
Tell me the Hours and Minutes, I implore,
Or I ſhall never ſee my dear *Eliza* more.

Anſwer. *One Day as I was in a muſing Vein,*
My roving Fancy did me entertain
With thoughts of Love, which did the Time beguile,
And great Delight attended me the while.
Straitway I ſtruck into a verdant Plain
To meditate, and eaſe my troubled Brain;
There I eſpy'd a Grove, moſt rich of Shade,
Where wanton Birds harmonious Muſick made:
All Things rejoyc'd in the moſt lovely Green,
Nature, in full Perfection, here was ſeen.
I plac'd myſelf under a ſpreading Tree,
Methought it was a charming Canopy.
Under this pleaſant Shade Eliza *ſweet,*
And her dear Lover, did together meet;
Where he oft lay encircled in her Arms,
In Love-Tranſports, amidſt a thouſand Charms,
But envious Chance did ſeparate theſe Friends,
Who hope next Meeting will make them Amends.
Ah! cruel Fate, to croſs poor Lovers ſo,
Methinks I feel the Pain they undergo,
Caus'd by that Traytor's Abſence, in each Breaſt
With Sighs and Tears which cannot be expreſt.
But leſt a Diſappointment he ſhould find
In their next Meeting, which they both aſſign'd,
To eaſe his Pain it doth my Pity move,
Who know the Pangs of diſappointed Love.
In his Eliza's *Arms let him be bleſt,*
And in her pretty panting Boſom reſt;
The Time is ſhort, which if they chance to miſs,
It puts an End to all his Earthly Bliſs;
Like a dear Lover, Love my Mind inſpires,
And his hard Caſe a Tenderneſs requires:
How long it is e're he Eliza *met,*
In this moſt pleaſant Shade, I find complete

To be one Hour, with Minutes forty-four
And Seconds fifty-ſix, nor leſs nor more.

65. My Grandſire and my Great Grandſire,
I'm ſure, conceived were
Both in one Inſtant, at one Time
Firſt breathed in the Air.
My Grandſire did no ſooner breath,
My Father did the ſame,
Great Grandſire my Father begot
Beſore my Father came
Into this World, where he did meet
With Troubles not a few,
My Family it did decreaſe,
I did the ſame renew.
My Grandmother and Siſter too,
Conceiv'd me, both as one:
My Aunt and Siſter, in like Caſe,
Brought forth another Son.
Tho' Grandmother and Mother too,
Did bear me as a Son,
My Grandmother transformed was,
Before my Life begun;
Though thus transformed, ſuffered Death,
Joſephus doth relate,
Hundreds of Years after my Birth,
He that ſaw it in that Eſtate.

Anſwer. *Your Paradox I muſt commend,*
Ingenious Mr. Lover,
And though 'tis difficult, intend
Solution to diſcover.

'Tis Moab, *who was Son of* Lot,
Your Paradox muſt anſwer,
Which of his Daughter was begot,
If this won't do, I can't, Sir;
Lot, *Grandſire and Great-Grandſire was*
To Moab, *and to his Brother,*
From hence, I think it comes to paſs,
'Tis true you mean none other;
No Wonder they conceived were,
(His Sire and Grandſire too)
Both in one Inſtant, breath'd i' th' Air
Together, as you ſhew,
Great Grandſire Haran *did beget*
His Father, Lot, *we read,*
In Uz *of the* Chaldees, *he yet,*
Long e'er Moab's *Birth, was dead:*
Thus in this World, poor Lad did meet
With Troubles manifold,
Sodom *being burnt, he did retreat,*
And dwelt in a ſtrong Hold:
Laid in the Cave, being drunk with Wine,
Begot Moab *and his Brother,*
Of's Daughter, who as you define,
Was Siſter and Grandmother.
Thus Grandmother and Mother too,
Did bear Moab *as a Son,*
His Grandmother, Lot'*s Wife, was dead,*
E'er Moab'*s Life begun;*
She was transformed into Salt,
The Scriptures plainly ſay,
Looking to Sodom *was her Fault,*
She God did diſobey;

This Pillar fixed on the Earth,
Joſephus *doth relate,*
Hundreds of Years after Moab's *Birth*
He ſaw't in the ſame State.

66. *Chriſtians* the Week's firſt Day for *Sabbath* hold,
The *Jews* the ſeventh (as they did of old)
The *Turks* the ſixth (as I have oft been told)
Now, Good Sir, pray tell to me,
How it is poſſible this Thing can be,
That ever a *Chriſtian*, *Jew* and *Turk*, theſe three,
Being altogether, in one Place may,
In and upon one and the ſelf ſame Day,
Have each his own true *Sabbath*, tell I pray.

Anſwer. *From the Place of the* Jew's *Abode let the other two ſet out,*
The Chriſtian Eaſt, *the* Turk Weſt, *and ſail the Globe about;*
Then with the Jew *they will agree when they again do meet,*
And all upon the Saturday *will their true Sabbaths keep.*

67. Walking one Evening on the Sandy Shore,
Where Shell-Fiſh breed, and Seas for ever roar,
I ſaw a Ship dance on the rolling Tide,
With curling Smoke advancing from her Side;
Which for ſome Moments my dull Senſe employs,
Before I heard the thundering Cannon's Noiſe.
Then from my Fob Time's Regiſter I drew,
And from her Sides a ſecond Lightning flew;
Full fourteen Seconds of ſwift Time expir'd,
Before the Sound my liſt'ning Ears admir'd.
Ye learned Heads, that Sound's Progreſſion know,
My Diſtance from the floating Veſſel ſhow.

Anſwer. The Diſtance of the Ship from you was three *Engliſh* Miles, 148 Feet, allowing Sound to move 1142 Feet in a Second of Time.

Note, The Sound of a Bell, in the Receiver of an Air-Pump, when the Air is exhauſted, is hardly heard; nor a Gun on the Top of *Pico Teneriff*; but in an unexhauſted Receiver, a ſmall Bell may be heard many Paces off, and the further, the more the Air is compreſſed. Theſe prove that Sound depends on the Vibrations of the ſonorous Body on the Air.

But leſt I tire any with Verſe, I now return again to Proſe.

68. *There are three remarkable Places on the* Terraqueous Globe, *to whoſe Inhabitants all the Stars are viſible on three certain Nights of the Year.*

Anſwer. Under the Æquator; for at the firſt Part of the Night you ſee one Hemiſphere, or half of the Stars, which in twelve Hours Time intirely ſets againſt Morning, when the other half appears that was intirely hid the Evening before: Thus there are not only three remarkable Places, but all Places under or near the Æquator; the beſt Time is when the Nights are longeſt, if the Places be not directly under the Æquator.

69. *There is a certain Place on the Earth, above whoſe Horizon* Saturn *is fifteen Years; and there is another of a conſiderable Diſtance from that, which hath* Jupiter *above five Years without Setting above their* Horizon.

Anſwer. This Place is under or near the *Poles,* where *Saturn* continues without Setting near fifteen Years; as *Saturn* entered *Aries* about the Middle of *March,* 1702, and paſſed thro' the ſix following Signs, entering *Libra* the 20th of *September,* 1715, which amounts to near 15 Years; during which Time it may be ſuppoſed that *Saturn* was ſeen above the *Horizon* of the *North Pole:* Alſo the other Place (mentioned in the Paradox) of a conſiderable Diſtance, is the *South Pole* where *Jupiter* might have been ſeen from *March* 1707, till *March* 1714.

70. *It may be demonſtrated by the* Globe, *that the Sun, Moon, and ſeveral of the Planets, do not move exactly* 15 *Degrees hourly, from the* Meridian *of ſeveral Places on the Continent of* America, *but come later to their* Meridian *than the preceding Day.*

Anſwer. Either it may be ſome floating Iſland; or this may ariſe from the diurnal Advance of the Sun, Moon and Planets in their particular Orbs, which exceed the 15 Degrees hourly, or the 360 Degrees of the Equinox by the ſaid diurnal Motion of the Planets.

71. *There is a certain Wall in the City of* London, *whoſe Situation is due* Eaſt *and* Weſt; *yet, 'tis certain, the* Sun *will ſhine on the* North *Side thereof before Six in the Evening, all the* Summer, *and on the* South *Side before Six in the Morning.*

Anſwer. The Wall here meant, whoſe Situation is due *Eaſt* and *Weſt*, the *North* Side is perpendicular, the Sun having *North* Declination all the *Summer*, will ſhine on the *North* Side before Six in the Evening; and the *South* Side of that Wall hath a reclining Plain, whoſe Reclination is greater than the Latitude of the Place; then may the Sun ſhine on that Plain before Six in the Morning all the *Summer.*

Or thus,

The Sun being a great Body, and upon Motion, ſome of it will paſs by the Wall before the Whole, and conſequently ſend ſome of its Rays on the North Side before it be exactly ſix o'Clock; which on the Equinoctial Day is preciſely when the Sun's Center is in a right Line with the Wall, which is a conſiderable Time after Part of it ſhines on the North Side, *i. e.* ſo long as the North Semidiameter of the Sun is tranſiting that Line of due Eaſt and Weſt.

72. *There are ſeveral remarkable Places, above* 100 *Miles aſunder, which have no*

Antæci; *and their* Periæci *are as far off them as their* Antipodes: *There are alſo two Places above* 500 *Miles aſunder, which have no* Periæci, *and their* Antipodes *are as near them as their* Antæci.

Anſwer. Thoſe ſeveral Places 100 Miles aſunder, are all on the Æquator, which therefore have no *Antæci*, and their *Periæci* are diametrically oppoſite, and are alſo conſequently *Antipodes.* The other two Places above 500 Miles aſunder, which have no *Periæci*, are the two *Poles*, who are both *Antiæci* and *Antipodes* to each other.

73. *'Tis certainly Matter of Fact, that three certain Travellers went a Journey, in which, tho' their Heads travell'd full twelve Yards more than their Feet, yet they all returned alive, with their Heads on.*

Anſwer. According to the Corollary of the laſt Propoſition of the third Book of *Whiſton*'s *Euclid. If any one ſhould travel over the whole Circumference of the Earth, the Way gone over by his Head would exceed that which was gone over by his Feet, by the Difference of Circumference; or by the Circumference of a Circle, whoſe Semidiameter is the Man's own Stature.*

Now, ſuppoſe *Drake*, *Cavendiſh*, or *Dampier*, each of them to be two Yards, then will the Diameter be four Yards, and the Circumference above twelve Yards; whereas as did they ride or go on the Top-Maſt, there would be ſtill a greater Difference in each of their Voyages round the Globe.

74. *Though it be generally affirm'd, that the Arch of a great Circle, betwixt two Places, is their ſhorteſt Diſtance; yet there are two Places ſo ſituated, as their Diſtance is conſiderably ſhorter than the Arch of a great Circle; yea, and there be ſeveral Places, betwixt which 'tis their longeſt Diſtance.*

Anſwer. At the Bottom of the Ocean, ſome deep Cave, Valley, Mine, or ſome Places in the Earth, conſiderably below the Surface thereof; and for the latter Part, it muſt intend the *Antipodes*, which is the longeſt or greateſt Diſtance poſſible.

75. *There are certain Places in* North *Latitude, whoſe longeſt Artificial Day, is conſiderably longer, by ſome Hours, than the longeſt Artificial Day with them in the ſame Degree of* South *Latitude.*

Anſwer. The *North Pole*, or near it, becauſe the Sun ſpends above eight Days more in the Artificial Day under the *North Pole*, betwixt *Aries* and *Libra*, than in the Artificial Day under the *South Pole*, betwixt *Libra* and *Aries*.

76. *Any two Places, of a conſiderable Diſtance*

ſtance from each other, whoſe Bearing is truly known, and is not on one of the Cardinal *Points; yet if a Ship keep continually on the ſame Point, on which the Port bears off, tho' they duly rectify the* Compaſs, *and regard both Currents and Winds, it may be plainly demonſtrated by the* Globe, *that inſtead of hitting it, ſhe will be further and further off, and never will hit it on that Point.*

Anſwer. Such is the Difference between the Angle of Poſition and the Rumb by the Compaſs, that if the Ship ſteers by the Compaſs, on the ſame Point that the Port bears off, by the Circle of Poſition, it will never hit.

77. *There is a certain Place in* Great-Britain, *where when the Tide is in, one may ſee the Sheep feeding on a certain neighbouring Iſland; yet when the Tide is out, and the Water at loweſt, not one can be ſeen, tho' they be feeding there at the ſame Inſtant.*

Anſwer. The Place may be the Wharf of *Greenwick*, and the Iſland of *Dogs* over againſt it, and the Appearance cauſed by the Refraction of Sight when the Water is high. *See Paradox* 18.

78. *That no Colours are real but what we call Red, Yellow, Green or Blue,* &c. *only appear ſo to us, according as Bodies variouſly receive*

receive their Light; and that there are no Colours in the Dark.

Anſwer. Nothing is ſo manifeſt to the Senſe as Colour, nothing ſo obſcure to the Underſtanding; which doubts whether it has a real Exiſtence, or whether it only appears ſuch to us, according as Bodies variouſly receive the Light; for we judge of them otherwiſe in the Twilight, in the Sun, and in the Shadow, otherwiſe beholding them ſlopingly, directly, or thro' a colour'd Glaſs. Are any Colours fairer than thoſe of the *Rainbow* and yet they are no more real than thoſe of the Clouds: The Whiteneſs we behold in the milky Way, ariſeth only from the Light of many ſmall Stars. Pictures are apprehended well or ill drawn according to their Situation: Nay, the viſible Species are nothing elſe but Qualities ſtreaming from every terminated Body, which alter the Medium, filling the ſame with their Images which they diffuſe even into the Organ. Now Colours are the ſame, being Qualities which actually change and alter their diaphanous and illuminated Body: Or the Subſtance of Light itſelf, differently reflected from thoſe objected Bodies, as in the Caſe of Priſms, *&c.* And to explain this Paradox more fully, extend a Yard of Scarlet Cloth, in an uniform Light, then fold it, and view one half in the Sun ſhine, the other half in the reflected Light, and fold it in and out, like the Paper Lanthorns of a Kite, all which diligently obſerve, as you move it thro' the various Degrees of Light and Shade, and you will admire the vaſt Variety of Colours apparent thereon, none of which are more real and inherent one than the other, only a Diſpoſition to reflect the Varieties of Reds, as lively Carnation, Sanguine, Murry (or ſable Black in the dark Shade or Abſence of Light) and this Diſpoſition ariſes from the Cloth being ting'd in a certain Liquor, whoſe minute Particles are, by reaſon of their Figure and Poſition, adapted to refract the incident Rays of Light after ſuch Modification, as to preſent Red, rather than Green, Blue, *&c.* which requires different Degrees of Refrangibility. *See* Newton's *Optics.*

79. *No Man can ſee the ſame Particle of an Object with both Eyes at once, nay not with the ſame Eye, if the Level of its viſive Axis be changed.*

Anſwer. This Paradox ſeems confirm'd both by Experience and Reaſon, 1. Becauſe no Man can make the Viſion of both his Eyes equally perfect; but beholding a Thing firſt with one Eye, the other being cloſed, ſhall conſtantly diſcover it to be greater in Dimenſions, in the Apprehenſion of one

Eye than of the other. So *Gaſſendus*, Epiſt. 2. Sect. 17. teſtifies that the Characters of his Book appear'd to his right Eye, by one fifth Part, greater in Dimenſions, tho' ſomewhat more obſcure, than to his left. 2*dly*, Becauſe of all twin Parts of the Body, one is always more perfect and vigorous than the other, which, if not the Effect of Cuſtom, may proceed from a more liberal Afflux of Spirits to one than the other: or the organical Conſtitution may vary a little, as if one Pupil be wider, the Chriſtalline more Convex, *&c.* Such like Cauſes neceſſitate a reſpective Diſparity in the Action.

80. *The ſame Object ſpeculated by the ſame Man, at the ſame Diſtance, and in the ſame Degree of Light, doth always appear greater to one Eye than the other*; *and that all Men ſee diſtinctly but with one Eye at once, contrary to that Axiom, That the viſive Axes of both Eyes concur and unite in the Object.*

Anſwer. Notwithſtanding the Repugnancy of this Paradox to common Opinion, yet its Verity ought to be aſſerted; for the Axes of the Eyes were ſo ordain'd by Nature, that when one is extended the other is relaxed, nor can they be both employ'd at once, tho' both at once may be relaxed, becauſe of the Paralleliſm of the Motion of the Eyes, which is evident to Senſe: Nor is there more

Neceſſity to uſe both Eyes, than both Arms or Legs at once; for Inſtance, look but at the Top of your own Noſe, and you will be ſoon convinc'd that you cannot diſcern it with both Eyes at once, but the right Side with the right Eye, and the left Side with the left Eye; for when the *Axe* of the right Eye is converted to the right Side of the Noſe, the *Axe* of the left muſt be converted towards the left Ear. For a further Illuſtration, in the Chamber ſet up a Staff, retreat a few Feet, and fix your Head againſt the Wall, then ſee what Pane of Glaſs, or Part thereof the Staff hides from the right Eye, when the left is ſhut; alſo, without moving the Head, ſhut the right Eye and open the left, and ſee what Part of the Pane is then hid, which will be very different from the former, and their Diſtance a ſunder is called the Parallax of Sight; then open both Eyes, and view if the *Axes* of both did meet and unite in the Staff, as is generally ſuppoſed, then, of Neceſſity, would you obſerve the Staff to eclipſe either both Parts of the Window together, or the Middle of the *Parallax*; but you ſhall obſerve it to do neither, but only one of the Parts, and that on which you ſhall fix one of your Eyes more intently than the other: Hence we may, with *Gaſſendus*, aſſure the Gunners that they ſhall ſhoot as right with both Eyes open as only with one: Thus in confus'd and imperfect Viſion; tho' it may be truly ſaid, a Man doth ſee with both Eyes at once, but not diſtinct and perfect, there being ſeveral Degrees of Viſion.

81. *That the Matter of a Body, when rarefy'd, doth poſſeſs no more of true Place than the Matter of the ſame Body condenſed.*

Anſwer. When a Fleece of Wool is diſtended, we ſay 'tis made more rare, and when compreſt more denſe; now the Rarefaction conſiſts in this, that the Hairs formerly united are diſunited, and the Spaces betwixt them become larger, in which no Particle of Wool is contain'd, and Denſity is quite the contrary; now tho' a Fleece of Wool expanded includes a greater Capacity therein than when compreſt, yet the ſingle Hairs thereof take no greater Space in either Capacity; for no Hair can poſſeſs more Space than its proper Bulk requires, but the empty airy Spaces intercepted, are enlarged in the Rarefaction and diminiſhed in the Compreſſion: Hence 'tis true, that the Matter of a Body rarefy'd, *i. e.* the ſundry indefinite Particles poſſeſs no more of true or proper Place than the Matter of the ſame Body condenſed, tho' the Surface of the ſame Body may poſſeſs more Place when rarefied than when condenſed, becauſe then it not only takes in the Particles of Space occupied by the Particles of Matter rarefied, but alſo all the enlarg'd Vacuities interſperſed.

82. *A certain Traveller, of unqueſtionable Integrity relates, he, in the* Eaſt-Indies, *ſaw Sun, Moon, and Stars all at once, at Noon-Day: Yea, and in the Reign of* Tiberius, *the Sun was ſeen to ſet at Noon, when it was Full-Moon; and tho' there was Darkneſs above the Earth, yet was there Light under it, nay, that very Darkneſs became a Light, and in a few Days two real Suns, not mock ones, aroſe in one Day, within a few Hours of each other.*

Anſwer. This conſiſts of ſeveral Parts, as to the firſt, Father *Kercher*, in his Deſcription of the ſubterraneous World, *Lib.* 2. tells us, that by the Help of a Teleſcope we may perceive the Sun, a Body of Fire unequal in its Surface, compos'd of ſeveral Parts of a different Nature; ſome fluid, ſome ſolid, and that his Diſk is a Sea of Fire, wherein one may perceive an eternal Agitation of the Waves of Flame, that in ſome Parts of it there ariſes a deal of Smoke: Now 'tis very probable, that a vaſt Quantity of Smoke might intercept and hinder the Brightneſs of the Sun; as in 1547, from the 4th to the 28th of *Auguſt*, the Sun appeared reddiſh, and not ſo bright as the Moon in her total Eclipſe, ſo that then there were many Stars viſible at Noon-day, which being as bright as at other Times, plainly ſhow, the fix'd Stars receive not their Light from the Sun, but are rather ſo many Suns themſelves, at an indefinite Diſtance.

As to the latter Part of the *Paradox*, becauſe the Reign of *Tiberius* is mention'd, probably that preternatural Darkneſs which happen'd at our Saviour's Crucifixion, which was then Full-Moon, is meant; and the Darkneſs began at Noon, or 6th *Jewiſh* Hour, and continued to the 9th, which miraculous Darkneſs alſo, proved a Light to *Dyoniſius* the *Areopagite* of whom it is reported, that beholding it, he ſaid, *Either the World is at an End, or the God of Nature ſuffers*; and in a few Days, *i. e.* on the third Day early in the Morning, before Sun-riſe, aroſe the very *Sun of Righteouſneſs*, who was, and is, *The true Light that enlightens every Man coming into the World*; according to *John* I. 9.

83. *There are ſeveral Places on the* Continent *of* Europe, *where, at ſome certain Seaſons of the Year, if a Feather and Globe of Lead were let fall exactly together, the Feather will reach the Bottom as ſoon as the Lead; which Places, then, are of ſuch deteſt-*

able Quality, that if a Sparrow, *never ſo lively, ſhould chance to fly over it, would immediately fall down in a Swoon, which upon breathing on, in the Turn of an Hand, will revive and be as briſk as ever.*

Anſwer. In any Place where the Air-Pump is, and at any Seaſon when the Receiver is exhauſted of the Air within, ſuch ſtrange Effects will happen: and by *breathing upon, in the Turn of an Hand,* is intended the Re-admiſſion of the Air, whereby the Sparrow will revive again.

84. *On the* Continent *of* Aſia, *there is a certain Country, ſome of whoſe Inhabitants, though they live not many Hours, yet they breed and bear without Food or Care, and their Generations have continued many Ages, where alſo the very Deſart affords them vegetable Springs of Water.*

Anſwer. Navarette tells us of a Tree called the *Bejuco,* which twiſting about other Trees, with its End hanging downwards, and that Travellers cut the Nib off it, and preſently a Spout of Water runs out from it as clear as Chryſtal, enough and to ſpare for ſix or eight Men. *I drank* (ſaith he) *to my Satisfaction of it, and found it cool and ſweet, and would drink it as oft as I found it in my Way; it is a Juice and natural Water. 'Tis the common Relief of the Herdſmen on the Moun-*

Mountains, when they are thirſty they lay hold on the Bejuco *and drink their Fill.* Collect. of Voyag. Vol. I. Pag. 355. This then is a delectable vegetable Spring of Water. Alſo if an Inſect may be term'd an Inhabitant, *Swammerdam* mentions the *Ephemeron*, which is both an unuſual and ſpecial Inſtance of the Brevity of Life, and a wonderful Inſtance of the ſpecial Care and Providence of GOD in the Conſervation of the Species of that Animal: For, 1*ſt*, As an Animal, whoſe Life is determined in about five or ſix Hours, it needs no Food. 2*d*. As to its Generation, in thoſe five Hours of its Life it performs that and other neceſſary Offices of Life; for in the Beginning of its Life it ſheds its Coat, and that being done, and the poor little Animal thereby rendered light and agil, it ſpends the reſt of its ſhort Time in friſking over the Waters, and, at the ſame Time the Female droppeth her Eggs on the Waters, and the Male his Sperm on them, to impregnate them; theſe Eggs are ſpread about by the Waters, and deſcend to the Bottom by their own Gravity, and are hatch'd by the Warmth of the Sun, into little Worms, which make themſelves Caſes of the Clay and feed on the ſame, without any Need of Parental Care.

85. *'Tis as demonſtrable as wonderful, that Mathematicians may draw Lines both ſtrait and circular, which ſhall always approach nearer and nearer to a given Line, even unto Infinity; but either will never be join'd to it, or if join'd will never unite or meet, ſave in one ſingle Point; and tho' the Angles formed at the ſaid Point may be diminiſhed by Arches, yet cannot ſo much as be biſſected by any right Line whatever; and the ſaid Angles are both greater, equal to, and leſs than a right Angle.*

Anſwer. The Corollaries of the 16th Propoſition of the third Book of *Whiſton*'s *Euclid*, aforeſaid, will illuſtrate this *Paradox*, to which I refer, when I have told my Reader that this *Paradox* means, not only the Angle of Contact, but the Approach of the Aſymptotes to an Hyperbola, both infinitely produced into a Diſtance, leſs than any given one, yet never concurring with it: By this Propoſition, is plainly inferr'd the infinite Diviſibility of Matter, Lines and Points.

86. *There is a certain Iſland on the* Continent *of* Europe, *ſome of whoſe Inhabitants are of ſuch exquiſite Sight, that even with one of their Eyes, they can actually behold ten Moons, real and true, all at once, above their* Horizon, *yea, and ten Times the Number of Stars beheld by others with both their Eyes at the ſame* In-

Inſtant: Yea, ſuch wonderful Properties have theſe ſingle-ſighted few, that in Things ſmall and near, which eſcape the Sight of their Neighbours, they behold new Worlds of Wonder, ſee all the diſtinct Parts and Functions of thoſe minute Animals: Nay, like Cats, they can ſee in the Dark, and in an Inſtant, and in the Dark alſo, draw, to the Life, and paint in lively Colours, far beyond Michael Angelo, *or any other, in the beſt Light and longeſt Time. Laſtly, ſuch dextrous Hands have they, that in the Turn of an Hand they can command the Sun, Moon, and Planets to riſe and ſet and perform their diurnal Revolutions and their annual ones in a few Minutes, and predict their mutual Eclipſes for many Years to come.*

Anſwer. Any Iſland, any where, provided there be Mathematicians furniſhed with exquiſite Teleſcopes, which will ſhew the five Satellites or Moons of *Saturn*, the four Moons of *Jupiter*, and our Moon, all which are ten in Number, ſuch will ſee through them ten Stars for one beheld by our Eyes, not aſſiſted with the like; nay, in the Milky-Way, where we can't behold one, they will ſee an innumerable Company ſo cloſe and thick ſet, that their united faint Light cauſe that Whiteneſs which gives it the Denomination aforeſaid. The *Middle* Part of the *Paradox* reſpects the Wonders of the Microſcope, and the next intends the admirable and inſtantaneous Painting to the Life, perform'd by the *Camera Obſcura* (or dark Room) of the Philoſophers; and the *Laſt* means the noble and uſeful Invention of the *Orrery*, which ſhews by one Turn of the Hand, the viſible Revolution of the *Earth*, *Moon*, *Venus* and *Mercury*, their mutual Aſpects, Eclipſes, *&c.* for any Time paſt, preſent or to come.

The Periodical Revolution of the Moons of Saturn *and* Jupiter.

		D.	H.	M.	S.		D.	H.	M.	S.
1	next *Saturn*	1	21	18	31	next *Jupiter*	1	18	28	00
2		2	17	41	27		3	13	14	00
3		4	13	47	16		7	3	43	00
4		15	22	41	11		16	16	32	00
5		79	7	53	57					

87 *There is a certain Iſland in* Europe, *for which, in four or five certain Months in the Year, if the ableſt Aſtronomers the World has, would calculate the Moon's Riſing, they would not only differ in Minutes and Seconds, but whole Hours; and from which if they take a Journey in one Part, their Shadows will diſappear; in another they will circuit all Parts of the* Compaſs.

Anſwer. This may reſpect the Variety of Riſings of the Planets and Stars, as to one the Coſmical, the other the Acronical, the third the Heliacal, the fourth the true or apparent diurnal Riſing of the Moon: Each Aſtronomer, tho' exactly true, reſpecting that particular Riſing, will differ from the reſt, not only in Minutes and Seconds, but Hours and Days: The latter Part of the *Paradox* reſpects the Diverſity of Shadows; under the Poles or polar Circles the Shadows move all round about, under the *Tropics* no Shadow at all when the Sun is in the *Zenith*; but in the *North* Temperate, our Noon Shadow is always *North*, and in the *South* Temperate, always *South*.

88. *To ſeveral Parts of this* Globe, *there be certain Planets, which are ſo far from coming to an Oppoſition, that they form neither* Trine, Square *nor* Sextile *Aſpect with the* Sun: *And there be other Places, where, if you bring a* Mariner's Compaſs, *tho' never ſo well touched and rectified, the Needle is of no Service, becauſe it will turn indifferently, to any Point of the* Compaſs, *tho' neither Iron nor Loadſtone be near.*

Anſwer. Venus and *Mercury*, whoſe Orbits are contain'd within the Orbits of the Earth, never form ſo much as a Sextile Aſpect with the Sun: And in or near the two *Poles* of the Earth, the beſt Needle turns indifferently to any Point of the *Compaſs*.

89. *There are ſeveral Planets or wandering* Stars, *which at certain Times, appear and diſappear, whoſe Light decreaſes as they come towards*

wards the Earth, and increaſes as they go from the Earth; yea, ſo tranſparent are they, that the ſmalleſt Stars *may be ſeen through them: Yea, and there be others ſo opake, that in Conjunction with the* Sun, *they appear as Spots in his Face, and the further they go from us, the bigger they appear.*

Anſwer. The firſt Part reſpects Comets which wander thro' all the Orbits of the fixed Stars, and do decreaſe in Light as they come towards the Earth, and increaſe as they go from it; thro' whoſe Tails the fixed Stars of the 6th Magnitude may be ſeen: The other opake Planets, which appear as Spots now and then in the Sun's Face, are *Venus*, *Mercury* and the *Moon*, which appears bigger in the Horizon when further from us, than in the Meridian when ſhe is above 3000 Miles nearer us, as in *Paradox* 19. The Cauſe of which *Phænomenon* ſome attribute to the Refraction, others to the Picture in the Bottom of the Eye, others to the Judgment in great Diſtances, ſays *Graveſand*, *Vol.* 2. *N.* 731. ‘ If we look on known Objects we judge from the apparent Magnitude and Colour which depends upon the Bigneſs of the Picture in the Bottom of the Eye, and this on the Angle under which the Object is ſeen."

Now 'tis well known to Aſtronomers, the Picture of the Sun in the Bottom of the Eye, is the ſame in both Caſes, and that of the Moon is leſs when it appears near the Horizon, becauſe ſhe is farther off by a Semidiameter of the Earth, which bears ſome Proportion to her Diſtance from the Earth, tho' little with reſpect to the Sun's Diſtanee from us, yet they appear greater near the Horizon by reaſon of the Interpoſition of the Fields and Houſes betwixt us and that Part of the Heavens: Alſo they appear nearer to us, and of a different Colour, becauſe of the Dimneſs, Groſſneſs, and Denſity of the lower Atmoſphere, the Body of the Air is twelve times greater near the Horizon than near the Zenith, through which the Rays of the Sun paſs; this not only magnifies the Refraction, but ſo dims its bright Rays, that without hurting the Eyes we can look upon him Riſing or Setting; but not ſo when on the Meridian near the Zenith; becauſe its Rays hitting againſt an infinite Number of Particles in the Air, near the Horizon are reflected or abſorb'd.

Howbeit, if we ſee the Bodies abovementioned thro' a Tube, this apparent Diſtance vaniſhes, as alſo the Magnitude which is deduced from it: From our Childhood upwards, and ſo continually, we join the Idea of Diſtance with the Increaſe of apparent Magnitude, whereby the Ideas are ſo cloſely joined that they cannot be ſeparated, not even in thoſe Caſes in which we know they lead us into Error.

Moreover, Refraction cauſes the horizontal Sun and Moon to appear of an oval Figure; for their inferior Limbs are more refracted and raiſed higher than their ſuperior Limbs are; and therefore theſe two Limbs will ſeem nearer to each other, while the Ends of the horizontal Diameter being equally refracted, keep the ſame Diſtance.

The *French* obſerved, that an Object which at Break of Day appeared in our Horizon or level Line, or a little above it, a little after Sun-riſe, appear'd below it; and the ſame Appearance they found in the Evening; for the Object before Sun-ſet appeared below the Level, which ſoon after Sun-ſet would appear in it inſomuch that in about half an Hour's Time the Difference has been obſerved to be no leſs than three Minutes, the Cold of the Night condenſing the Vapours, making them of different Denſity, cauſes the Refraction aforeſaid. Nay, three Points which appear at one Time in a ſtrait Line, have at another Time, within half an Hour, appear'd out of it conſiderably. Hence, even the Sight itſelf that is the Ray of Light, paſſing from Point to Point through the Air, is not a ſtrait Line, as to the Poſition, by reaſon of its different Refraction which is in the Medium of the Air.

90. *There be many Places of the Earth, to whoſe Inhabitants the* Sun *and above twice ſeven Planets, may be all viſible in one certain Night of the Year; and one of the ſaid Planets, tho' it ever changes, yet ſtill, to them, turns always the ſame Face.*

Anſwer. The five Moons of *Saturn*, the four Moons of *Jupiter* and our Moon, which make ten Moons, beſides the uſual Planets *Venus* and *Mercury*, which are inferior, and *Saturn*, *Jupiter* and *Mars*, which are the ſuperior Planets, beſides the Earth. Howbeit, all theſe Moons are viſible only in thoſe Places where are extraordinary Teleſcopes, and only in thoſe Nights when they are out of the Shade of their primary Planets, about which they roll. 2*dly*, Our Moon, though ever changing, as to her Phaſes continually increaſing or decreaſing in Light; yet becauſe her periodical Revolution, about her Orbit and Axis, is perform'd exactly in the ſame Space of Time; therefore the Moon's Axis keeps its Paralleliſm, and ſtill obverts the ſame Hemiſphere or Face to the Earth, which as to its Illumination and Obſcuration, is continually changing, according to its Receſs from and Exceſs to the Sun.

91. *There is a certain Fire which burns at a Diſtance, and yet thoſe that are ſeveral Yards nearer and between, are not ſo much as warm'd*

by it; nay, its Heat is greater the further it is from us, and the leſs the nearer it is to us.

Anſwer. This muſt be the Sun, which appears like a burning Ocean, whoſe Beams ſcorch and burn near the Earth, at the Foot of a Mountain; yet on the Top of *Pico Teneriff*, which is many Yards nearer to it, the Sun's Reflection ſcarce reaches or warms; where the cold Air continually changes for freſh. Moreover, the Sun's Heat, every Body knows, is greateſt in Summer, when he is fartheſt from us, and leaſt in Winter when he is neareſt to us; for the Degrees of Heat and Cold do not altogether depend upon our Diſtance from the Sun, the Difference of which Diſtance is but ſmall, and bears little Proportion to the Sun's Diſtance from us; but there are other powerful concuring Cauſes which have certain Effects. For firſt of all, the direct Force of the Sun's Rays, are much ſtronger than when they are receiv'd obliquely; becauſe there are much fewer Rays in the ſame Space, when the Rays fall obliquely than direct: Beſides in Winter, when the Sun is neareſt to us, as aforeſaid, his Rays or Beams paſs through a much greater Quantity of Air, and are deeper immers'd in our Atmoſphere than they are in Summer when the Sun is fartheſt off us. Again, in Summer we have 16 Hours Sun-ſhine, and but eight Hours Night, but in the Winter the quite contrary; therefore no Wonder that the longer any hard Body is expoſed to the Fire the hotter it ſtill grows; for the Action of the Sun, by which all Bodies are heated, is not tranſient, as its Illumination is, but permanent: For this Cauſe, 'tis hotter under the Tropics than the Æquator itſelf, becauſe the Days are longer by an Hour and half there than under the Line, from whoſe Zenith the Sun recedes three Degrees in three Weeks Time; but is about two or three Months going ſo far from the Zenith under the Tropics.

92. *There is one certain Place of the World where the Planets, both Inferior and Superior, may be beheld conſtantly to move forward, in the ſame regular and uniform Tenor; tho' to moſt Places of the Earth they appear at the ſame Time to be ſtationary, retrograde, or to move very unequally.*

Anſwer. At the Sun, the true Center of the World's Syſtem, the Planets move all regular and direct; tho' to us, out of the Center of their Orbits, they appear ſometimes direct, ſometimes retrograde, and ſometimes ſtationary.

93. *Of all the Stars in a Star-light Night, though their Numbers ſeem near infinite, and* FLAMSTEED *has given a Catalogue of* 3000, *yet*

yet it will be hard for a piercing Eye to reckon, at any Time above 100; *notwithſtanding in a clear Winter's Night, without Moon-ſhine, at firſt View they ſeem to be innumerable.*

Anſwer. This Appearance is only a Deception of our Sight, ariſing from their vehement and ſtrong Twinkling, while we look upon them confuſedly and without reducing them to any Order; yet that in Reality their Number is next to infinite, appears from the Diſcoveries the Teleſcope has made; for Dr. *Hook*, with only his 12 Foot Teleſcope, numbers 78 Stars in the Compaſs of the *Pleiades*, where only ſix appear to the naked Eye; and *Anton. Maria de Reita* affirms, he has number'd in the ſingle Conſtellation of *Orion*, 2000.

94. *Notwithſtanding the diurnal Motion of the Earth about its Axis, and the annual Motion of the ſame about its Orbit, yet have ſundry Aſtronomers attributed a third Motion to the Earth,* i. e. *that whereby the Parallelijm of its Axis is maintain'd, which others affirm is no Motion.*

Anſwer. The Paralleliſm of the Earth, or that conſtant Poſition of the Earth's *Axis*, which in every Point of its Orbit, is parallel to itſelf in any other Point, which never changes its Direction, but always looks to the ſame Point of the Heavens, doth neceſſarily follow; if the Earth hath no other Motion but that round the Sun and the other round its own Axis. For tho' all the Diameters of the Earth will conſtantly change their Poſition by this Rotation about its *Axis*, except the *Axis* alone, which will remain at Reſt (like the Center in a Circle) in its former State, the Points in the *Axis* being the only Points in the Body which have no Rotation.

95. *There are ſeveral Planets ſaid to be in Conjunction with the Sun, not only when they appear in the ſame Degree of their Orbit with the Sun, but when they are in that Degree of their Orbit diametrically oppoſite to the former.*

Anſwer. The ſeveral Planets are *Venus* and *Mercury*, who have a two-fold Conjunction with the Sun, both in the ſuperior and inferior or oppoſite Points of their Orbits; in the firſt they are like Spots between the Sun and us, and in the laſt the Sun is between us and them.

96. *There are ſundry Places of the World, when coldeſt much hotter than any Part of the* Torrid-Zone, *and yet no burning Mountain; and other Places, when hotteſt much colder than our* Frigid-Zone.

Anſwer. The firſt muſt be in *Venus* or *Mercury*, and the laſt muſt be in *Saturn* or *Jupiter.*

97. *What Seas have their Waters freſh, and no Tide neither ebbs nor flows? And what Straits and Springs have their Waters ſalt? and the Reaſons why?*

Anſwer. The *Dead Sea*, the *Euxine* and *Baltic* Seas have freſh Water and no Tide; and the Streights of *Davis*, *Hudſon*, and *Frobiſher* are ſalt, becauſe the *Baltic* Sea has many Iſlands at the Mouth of it, that it takes not in that Quantity of Water from the main Ocean, but receives more from the many freſh Water Rivers which diſcharge themſelves into it from *Poland*, *Ruſſia*, and *Sweden*; the like concerning the *Euxine*. And in *Burgundy* and *Lorain* there are Springs which ſend forth ſalt Water; becauſe they diſſolve the Salt which they meet with in the Earth as they run along.

98. *What is the Reaſon of the Cohæſion of two well poliſh'd Glaſſes or Marbles together ſo hard that three Quarters of an Hundred Weight, faſtened to the lower, will not ſeparate it from the upper one?*

Anſwer. Doubtleſs the Preſſure of the Atmoſphere, which preſſing hard upon the obverted Planes of the Glaſs or Marble; and there being no Air between, to reſiſt the external Preſſure, there muſt neceſſarily follow the Cohæſion aforeſaid: For in an exhauſted Receiver one Pound Weight ſufficed to part two ſuch Marbles, that no leſs than 80 Pound Weight, in an open Air could ſeparate them. This Experiment of the famous *BOYL*, confirms the Preſſure of the Atmoſphere, and the above Solution.

99. *'Tis reported by the Duke of* Genoa, *that a Ship was found in* Switzerland *with the Bodies of forty Men in it: Alſo another Ship, in the Bottom of a Lake in* Italy, *which laſt was ſuppoſed to be ever ſince the Reign of* Tiberius: *How got they there?*

Anſwer. That the firſt could not be from *Noah*'s Time, becauſe Navigation, eſpecially on the Ocean, was not grown to ſuch a Perfection in his Time, as to make Ships of that Bigneſs, and Anchors of that Perfection. 2*dly*, If ſuch Ships had been, it might probably happen that ſome other Men or Creatures might have eſcaped with Life, beſides thoſe in the Ark. 3*dly*, How could a Ship of that Bigneſs, be carried 600 Feet under Ground? Certainly 12 Months, much leſs 40 Days, Rain could not reduce the Land to ſuch a Quagmire.

Next, A ſubterraneous Navigation ſeems a ridiculous Suppoſition; wherefore, for aſſigning a Cauſe ſufficient, I conceive there cannot be a more probable one than the Effect of Earthquakes. Suppoſe then that Part of *Switzerland* was the Bottom of the Sea, when this Veſſel was directly over this Place, and an Earthquake happen'd juſt underneath, which did raiſe the ſame above the Level of the Water as much as 'tis now, and cloſing the Gap, incloſed both Water, Ship, and Men, which it had ſwallowed.

100. *That there is another more powerful Cauſe of the Twilight than the Reflection of the Earth's Atmoſphere.*

Anſwer. There is an Æthereal Air or Atmoſphere round about the Sun, which ſhines after the Sun's Body is ſet, this Orb of the Sun's Atmoſphere riſing ſooner and ſetting later than the Sun itſelf, ſhines out at Mornings and Nights in a circular Figure, it being a Segement of the Sun's Atmoſphere, cut by the Horizon, and its Light quite of another Sort than that which is made by the Reflection of our Atmoſphere, but its Duration is much ſhorter.

P. Nonnius, to find the Length of the Twilight, watch'd the Time after Sun-ſet, when the Twilight in the *Weſt* was ſhut in, ſo that no more Light appear'd there than in any other Part of the Sky near the Horizon: Then, by one of the known fixed Stars having taken the true Hour of the Night, found by ſeveral Nights Obſervation, that at the Time of ſhutting in the Twilight, the Sun was under the Horizon 18 Degrees;

and until the Sun was depreſs'd ſo low, the Twilight continued.

The Sun being in the *Winter* Tropic maketh the longeſt Twilight, and from thence, as the Days increaſe the Twilight decreaſes until it comes to the ſhorteſt, which is in a certain Parallel between the Tropic and Equinoctial, the Declination whereof is found by this Proportion: As Co-tangent of the Latitude to the Sine of the Latitude, ſo is the Tangent of nine Degrees to the Sine of the Declination of the ſaid Parallel. But before the Crepuſculum or Twilight comes to be the ſhorteſt, there is another Parallel, in which the Crepuſculum is equal to the Crepuſculum of the Equinoctial, which is found by this Proportion: As Radius to the Sine of the Latitude, ſo is the Sine of 18 Degrees to the Sine of the Declination of the Parallel ſought.

PART III.

AN

APPENDIX,

Containing ANSWERS to the Hundred *Arithmetical Problems* left unanſwered in *HILL*'s ARITHMETICK, and *ALEXANDER*'s ALGEBRA.

1. TO *find a Number which being multiply'd by* 3, *ſubſtracting* 5 *from the Product, and the Remainder divided by* 2, *if the Number ſought be added to the Quotient, that the Sum may be* 40.

Let $b=3$, $c=5$, $d=2$, $f=40$, and for the Number ſought put a.

Then	1	$\frac{ab-c}{d}+a=f$ *per Queſtion*,
$1\times d$	2	$ab-c+ad=df$
$2+c$	3	$ab+ad=df+c$
$3\div b+d$	4	$a=\frac{df+c}{b+d}=17$.

2. *To find a Number, which being multiplied by* 12, *and* 48 *added to the Product, as much may be produced, as if the ſame Number ſought were multiplied by* 18.

Let $b=12$, $c=48$, $d=18$, and $a=$ the Number ſought.

Then	1	$ab+c=ad$
$1-ab$	2	$ad-ab=c$
$2\div d-b$	3	$a=\frac{c}{d-b}=8$.

3. *To find a Number to which if* 11 *be added and* 7 *ſubſtracted from the ſame Number* (viz. *the firſt*) *the Sum of the Addition may be double the Remainder.*

Let $b=11$, $c=7$, and $a=$ the Number ſought.

Then	1	$a+b=2a-2c$
$1+2c-a$	2	$a=b+2c=25$.

4. *To find a Number, to which if its double (triple, quadruple,* &c.) *be added, the Square of the ſame Number may be produced.*

For the Number ſought put a.

Then	1	$a+2a=aa$ *per Queſt.*
$1\div a$	2	$a=3$ Double
	3	$a+3a=aa$
$3\div a$	4	$a=4$ Triple.
and	5	$a+4a=aa$
$5\div a$	6	$a=5$ its Quadruple.

5. *To find a Number, which if added to itſelf, and the Sum multiplied by the ſame; and the ſame Number ſtill ſubſtracted from the Product: And laſtly, the Remainder divided by the ſame, that it may produce* 13.

Let $b=13$, and $a=$ the Number ſought.

Then	1	$2aa-a=$;
$1\div a$	2	$2a-1=b$ *per Queſt.*
$2+1$	3	$2a=b+1$
$3\div 2$	4	$a=\frac{b+1}{2}=7$.

6. *To divide the Number* 16 *into* 2 *Parts, ſo that the Square of the greater Part may exceed the Square of the leſs by* 32.

Let $b=16$, $c=32$, and a and $e=$ the two Parts.

Then	1	$a+e=b$	*per Queſt.*
	2	$aa-ee=c$	
$2\div1$	3	$a-e=\frac{c}{b}=2$	
$1+3$	4	$2a=b+2$	
$4\div2$	5	$a=\frac{b+2}{2}=9$ & $16-9=7=e$	

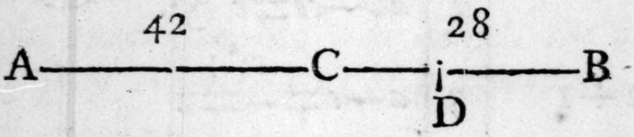
7. *To divide the Number* 36 *into* 2 *Parts, ſo that if* 12 *be added to the firſt, and* 6 *to the ſecond, the former may be the Double of the latter.*

Let $b=36$, $c=12$, $d=6$ and let $a=$ the Greater, then will $b-a=$ the Leſſer Number.

	1	$a+c=2b-2a+2d$ *p. Queſt.*
$1+2a-c$	2	$3a=2b+2d-c$
$2\div3$	3	$a=\frac{2b+2d-c}{3}=24$
and	4	$b-24=12=$ the Leſſer.

8. *Let the Line A B (of* 70 *Parts) be divided any how in C (ſo that AC may be*

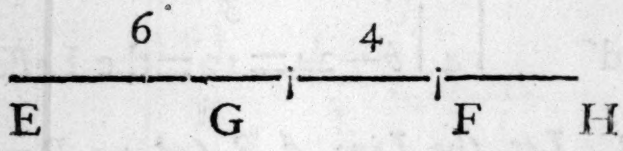
42, *BC* 28) *it is required to divide the ſame Line again in another Point:* For Example, *in D, ſo that the Rectangle ADC may be equal to the Square DB. Let the Segement CD be enquir'd (which being obtain'd AD, DB will be known.)*



Let $b=42$, $c=28$, and for the Segement *CD* put a, then will the Segement $D.B.=c-a$.

Then	1	$aa+ab=cc-2ac+aa$ per Q.
$1-aa+2ac$	2	$ab+2ac=cc$
$2\div b+2c$	3	$a=\frac{cc}{b+2c}=8$ and $28-8=20=D.B.$

9. *Let the Line EF be divided any how in G (ſo that EG may be* 6, *GF* 4) *it is required to produce this right Line EF (for Example unto H) ſo that the Rectangle EHF may be equal to the Square GH; the Length of FH is required.*



Let $b=6$, $c=4$, and the Line $FH=a$.

Then

Then	2	$aa+ab+ac=aa+2ac+cc$ per *Queſtion.*
$1-aa-2ac$	2	$ab-ac=cc$
$2\div b-c$	3	$a=\dfrac{cc}{b-c}=8.$

10. *A General diſpoſing his Army into a Square Battle, finds he has* 284 *Soldiers over and above; but increaſing each Side with one Soldier, he wants* 25 *Soldiers to fill up the Square: How many Soldiers had he?*

Let $b=284$, $c=25$, $d=1$, and for the Root of the Square firſt form'd put a.

Then	1	$aa+b+c=aa+2ad+dd$ per *Q.*
$1-aa$	2	$2ad+dd=b+c$
$2-dd$	3	$2ad=b+c-dd$
$3\div 2d$	4	$a=\dfrac{b+c-dd}{2d}=154$
4 @	5	$aa+284=24000$ his whole Army.

11. *A certain Captain ſends out* $\frac{1}{3}$ *of his Soldiers* $+10$; *there remains* $\frac{1}{2}+15$. *How many Soldiers had he?*

Let $b=10$, $c=15$, and for the Number he had at firſt put a.

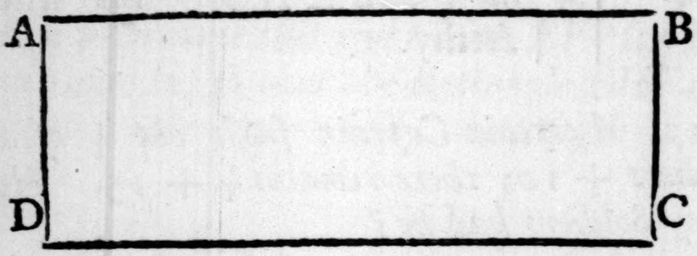
Then	1	$\frac{1}{3}a+b+\frac{1}{2}a+c=a$
1×6	2	$5a+6b+6c=6a$
$2-5a$	3	$a=6b+6c=150.$

12. *There is an Army to which if you add* $\frac{1}{2}$, $\frac{2}{3}$ *and* $\frac{3}{4}$, *of itſelf, and take away* 5000, *the Sum total will be* 100000. *What was the Number of the Army?*

Let $b=5000$, $c=100000$, and $a=$ the Army.

Then	1	$a+\frac{1}{2}a+\frac{2}{3}a+\frac{3}{4}a-b=c$ *per Qu.*
1×24	2	$\left.\begin{array}{l}24a+12a+16a\\+18a-24b\end{array}\right\}=24c$
$2+24b$	3	$70a=24c+24b$
$3\div 70$	4	$a=\frac{24c+24b}{70}=36000.$

13. *In the Rectangular ABCD, the Difference of the greater Side AB, and of the leſſer Side BC is* 12; *but the Difference of the Squares of the Sides* 1680. *What are the Sides of the Rectangle ABCD?*

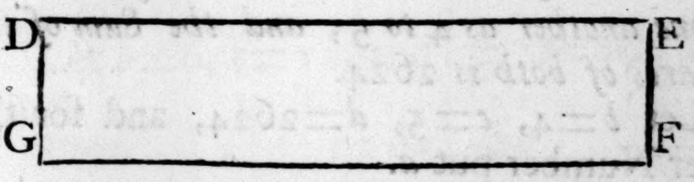


Let $b=12$, $c=1680$, and put $a=$ the leſſer Side, then will $a+b=$ the greater.

And

And	1	$aa+2ab+bb-aa=c$ *per Queſt.*
1,	2	$2ab+bb=c$
$2-bb$	3	$2ab=c-bb$
$3\div 2b$	4	$a=\frac{c-bb}{2b}=64$
and	5	$64+12=76$ the greater Side.

14. *The Length DE of the Rectangle DEFG, is twice the Breadth EF; and the Sum of the Squares of the Length and Breadth is ten Times the Sum of the two Sides DE, EF. What are the Sides of the Rectangle DEFG?*



Let $a=$ the leſſer Side, then will $2a=$ the greater.

And	1	$aa+4aa=10a+20a$ *per Queſt.*
$1\div a$	2	$5a=10+20$
$2\div 5$	3	$a=\frac{10+20}{5}=6$
and	4	$2a=12$ the greater Side.

15. *To find two Numbers in Proportion of* 2 *to* 3, *whoſe Product, if they be multiplied by one another ſhall be* 54.

Let $b=2$, $c=3$, $d=54$ and $a=$ the leſſer Number ſought.

Then	1	$b : c :: a : \frac{ac}{b} =$ the greater
and	2	$\frac{aac}{b} = d$ *per Queſtion.*
$2 \times b \div c$	3	$aa = \frac{db}{c}$
3 *uu*	4	$a = \sqrt[2]{\frac{db}{c}} = 6$
		2 : 3 : : 6 : 9 = the greater.

16. *To find two Numbers whoſe* Ratio *is to one another as* 4 *to* 5; *and the Sum of the Squares of both is* 2624.

Let $b=4$, $c=5$, $d=2624$, and for the leſſer Number put a.

Then	1	$b : c :: a : \frac{ac}{b}$
and	2	$aa + \frac{aacc}{bb} = d$ *per Queſtion.*
$2 \times bb$	3	$aabb + aacc = bbd$
$3 \div bb + cc$	4	$aa = \frac{bbd}{bb+cc}$
4 *uu*	5	$a = \sqrt{\frac{bbd}{bb+cc}} = 32.$
and	6	4 : 5 : : 32 : 40 = the greater.

17. *To find the Side of a Square, whoſe Area is to the Sum of the Sides in a given* Ratio, *as* 45 *to* 12.

Let $b=45$, $c=12$, and $a=$ the Side of the Square.

Then	1	$b:c::aa:4a$
and	2	$4ab=caa$ *per Queſtion.*
$2\div a$	3	$ac=4b$
$3\div c$	4	$a=\frac{4b}{c}=15.$

18. *To find the Side of a Cube, whoſe* Superficies *is to the Solidity, in a given* Ratio, *as* 6 *to* 11.

Let $b=6$, $c=11$, and $a=$ the Side of the Cube.

Then	1	$b:c:6a:aaa$
∵	2	$aaab=6ca$
$2\div a$	3	$aab=6c$
$3\div b$	4	$aa=\frac{6c}{b}$
4 *uu*	5	$a=\frac{\sqrt{6c}}{b}=11.$

19. *A certain Man hires a Labourer, on this Condition, that for every Day he work'd he ſhould receive* 12 *Pence, but for every Day he was idle he ſhould be mulct'd* 8 *Pence.*

When 390 *Days were paſt, neither of them were indebted to one another. How many Days did he work, and how many was he idle?*

Let $b=12$, $c=8$, and $a=$ the Number of Days he worked, then will $d-a=$ the Number of Days he play'd.

Then	1	$ab=cd-ca$ *per Queſtion.*
$1+ca$	2	$ab+ca=cd$
$2\div b+c$	3	$a=\frac{cd}{b+c}=156$
and	4	$390-156=234$ the Days he was idle.

20. *A certain Gentleman hires a Servant, and promiſes him* 24 *Pounds yearly Wages, together with a Cloak. At* 8 *Months End the Servant obtains Leave to go away, and inſtead of his Wages receives a Cloak* + 13 *Pounds. How much did the Cloak coſt?*

If from 24*l.* you take 13, the Sum he receiv'd, the Remainder 11 will be what was deducted for the 4 Months. Then as 4 : 11 :: 12 : 33 the Value of the whole Year's Service, from which if you take 24 the Remainder is the Value of the Cloak, *i. e.* 9*l.*

21. *A Perſon being aſked how old he was, anſwered, If I quadruple* $\frac{2}{3}$ *of my Years, and add* $\frac{1}{2}$ *of them* + 50 *to the Product; the Sum will be ſo much above* 100, *as the Number of my Years is now below* 100.

Put $b=50$, $c=200$, and $a=$ his Age.

	1	$\frac{8}{3}a+\frac{1}{2}a+b+a=c$ *per Queſt.*
1×6	2	$16a+3a+6b+6a=6c$
$2-6b$	3	$25a=6c-6b$
$3\div 25$	4	$a=\frac{6c-6b}{25}=36.$

22. *One being aſked what Hour of the Day it was, anſwer'd, The Day at this Time is* 16 *Hours long; if now* $\frac{1}{2}$ *of the Hours paſt be added to* $\frac{2}{3}$ *of the Remainder, you will have the Hour deſired, reckoning from Sun-riſing.*

Put $b=16$, $a=$ the Time paſt, then $b-a=$ the Remainder.

1×6	1	$\frac{1}{2}a+\frac{2b-2a}{3}=a$
2,	2	$3a+4b-4a=6a$
2	3	$7a=4b$
$3\div 7$	4	$a=\frac{4b}{7}=9\frac{1}{7}$ the Hours paſt, which added 4, the Sun's riſing gives 1 and $\frac{1}{7}$.

23. *From* Norimberg *to* Rome *are* 140 *Miles: A Traveller ſets out at the ſame Time from each of the two Cities, one goes* 8 *Miles a Day, the other* 6. *In how many Days from their firſt ſetting out will they meet one another, and how many Miles did each of them go?*

Here 'tis plain, the whole Diſtance muſt be divided by the Miles they both travelled in one Day, and the Quotient will ſhew the Number of Days which they perform'd their Journey in, which being multiplied by the Number of Miles each travelled in a Day, will give each of their Diſtance; ſo here if 140 be divided by $6+8$ the Quotient will be 10, which is the Number of Days, and $8\times10=80$, and $6\times20=60$ the Number of Miles each travelled.

24. *A certain Meſſenger goes ſix Miles every Day:* 8 *Days after, another follows him, and he goes* 10 *Miles a Day. In what Number of Days will he come up to the firſt?*

Here the firſt Meſſenger having gone 48 Miles before the Second ſet out, if that Number be divided by 4, the Number of Miles gained by the Second, the Quotient will ſhew the Number of Days to be travelled before the Firſt is overtaken, which in this Caſe is 12.

25. *A certain Meſſenger goes* 6 *Miles a Day; and after he has gone* 56 *Miles, another follows him who goes* 8 *Miles a Day. In how many Days will he come up to him?*

By the above Directions (this Queſtion being of the ſame Kind with the former) the Anſwer will be found to be 28.

26. *One bought 3 Books, whoſe Prices were in Proportion as* 12, 5, 1 : *If the Price of the firſt be doubled, of the ſecond tripled, of the third quadrupled; the Sum of theſe Products will as much exceed* 10 *Crowns, as the Sum of the Prices of the Greateſt and Middle is below* 5. *How much did the ſaid Books coſt?*

Put $b=12$, $c=5$, $d=1$, and $a=$ the Price of the firſt Book, $f=15$ Crowns.

$\because$	1	$b : c :: a : \frac{ac}{b}$
$\because$	2	$b : d :: a : \frac{ad}{b}$
	3	$2a+\frac{3ac}{b}+\frac{4ad}{b}+a+\frac{ac}{b}=f$ per *Queſt.*
$3\times b$	4	$2ab+3ac+4ad+ab+ac=bf$
$4\div 3b+4c+4d-$	5	$a=\frac{bf}{3b+4c+4d}=3$,
		then $\frac{ac}{b}=1\frac{1}{4}$, $\frac{ad}{b}=\frac{1}{4}$ the Prices required.

27. *Suppoſe the Number* 50 *were to be divided into two Parts, ſo that the greater Part being divided by* 7, *and the Leſs multiplied by* 3, *the Sum of this Product and the former Quotient, may make the ſame Number propoſed, which was* 50.

Let $b=50$, $c=7$, $d=3$.

	1	$\frac{a}{c}+bd-ad=b$ *per Queſtion.*
$1\times c$	2	$a+bcd-acd=bc$
$2\div cd-1$	3	$a=\frac{bcd-bc}{cd-1}=35$, which taken from 50 remains $15=$ the Leſſer.

28. *Let the Number* 20 *be divided into two Parts, ſo that the Square of the leſs Part being taken out of the Square of the greater, may leave the very Number propoſed, which was* 20 *(or may leave the double, triple,* &c. *of the Number propos'd.)*

Let $b=20$, for the greater Part, a then will the leſſer be $b-a$ the Square of the greater is aa, of the leſs is $aa-2ab+bb$.

Then	1	$2ab-bb=b$ *per Queſtion.*
$1+bb$	2	$2ab=bb+b$
$2\div 2b$	3	$a=\frac{bb+b}{2b}=10.5$ conſequently $95=$ the Leſs.

29. *If a Man gains* 30 *Crowns a Week, how much muſt he ſpend a Week to have* 500 *Crowns, together with the Expence of* 4 *Weeks remaining at the Year's End?*

In this Queſtion, find what he gains in a Year, then take therefrom 500 Crowns, the

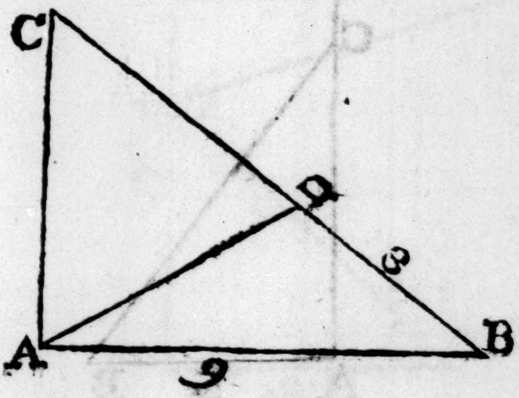
Remainder as he is to have 4 Weeks Expences to ſpare, is to be divided by 56, and the Quotient will be $18\frac{13}{14}$, the Sum to be ſpent each Week.

30. *A Labourer, after 40 Weeks in which he had been at Work, lays up 28 Crowns---the Pay of 3 Weeks; and finds that he had expended 36 Crowns + the Pay of 11 Weeks. What Pay did he receive a Week?*

Let $b=40$, $c=28$, $d=36$, and $a=$ the Crowns he received *per* Week.

Then	1	$d+11a+c-3a=ab$ *per Qu.*
1,	2	$ab-8a=c+d$
$2\div b-8$	3	$a=\dfrac{c+d}{b-8}=2.$

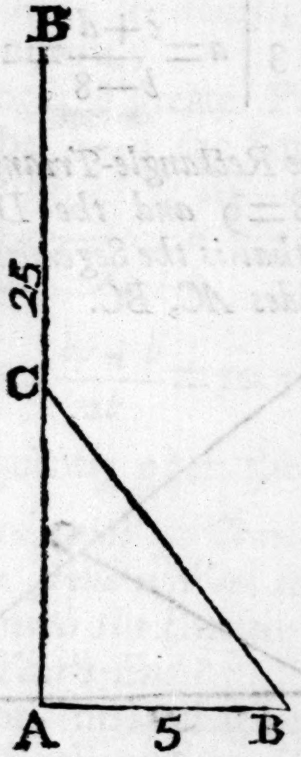
31. *In the Rectangle-Triangle ABC, is given the Baſis AB=9 and the Difference of the other Sides, that is the Segement BD=3. Required the Sides AC, BC.*



Let $b=9$, $c=3$, and $a=$ AC.

Then	1	$aa+bb=aa+2ac+cc$ *per* E. 47, 1,
$1-aa$	2	$2ac=bb-cc$
$2\div 2c$	3	$a=\dfrac{bb-cc}{2c}=12=$ AC
		conſequently BC $=15$.

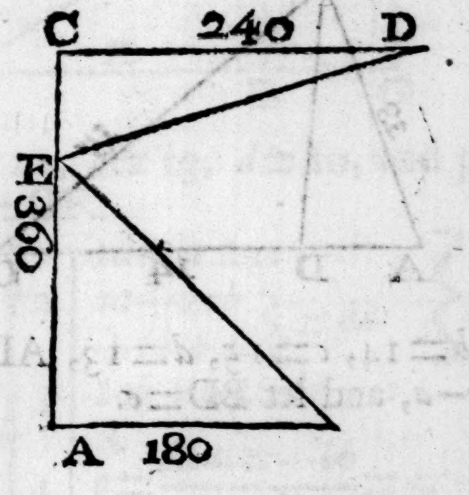
32. *In the Rectangle-Triangle ABC, is given the Baſis AB* $=5$, *and the Sum of the other Sides AC+BC* $=25$. *Ruquired the Sides AC, BC ſeverally.*



Let $b=25$, $c=5$, and AC$=a$ then will BC$=b-a$.

And	1	$aa+cc=bb-2ab+aa$ *per* E 47, 1
$1-aa+2ab$	2	$2ba+cc=bb$
$2-cc\div 2b$	3	$a=\frac{bb-cc}{2b}=12$
		which taken from 25 there remains 13$=$BC.

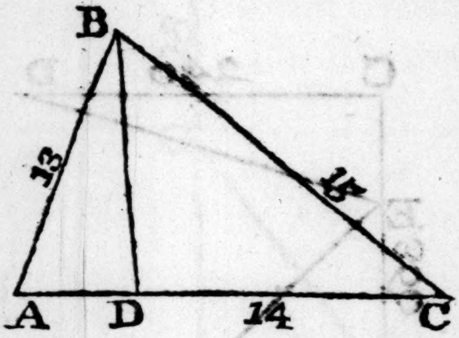
33. *Suppoſe two Towers, AB* 180 *Feet high, and CD* 240, *at the Diſtance AC* 360 *Feet. A Ladder is to be ſet upon the Line AC, at ſome Point, ſuppoſe in E, of ſuch a Length, as from thence it may reach the Top of both the Towers. We require the Point E in the Line of Diſtance, as alſo the Length of the Ladder EB, ED.*



Let $b=360$, $c=240$, $d=180$, and $a=$ CE, then will AE$=b-a$, and let $e=$ED the Length of the Ladder.

Then	1	$aa+cc=ee$ *per* E, 47, 1,
and	2	$bb-2ab+aa+dd=ee$
1, 2,	3	$aa+cc=\left\{\begin{array}{l}bb-2ab\\+aa+dd\end{array}\right.$
$3+2ab-aa$	4	$2ab+cc=bb+dd$
$4-cc\div 2b$	5	$a=\frac{bb+dd-cc}{2b}=145$
1 *uu*	6	$e=\sqrt[2]{aa+cc}=280.4014.$

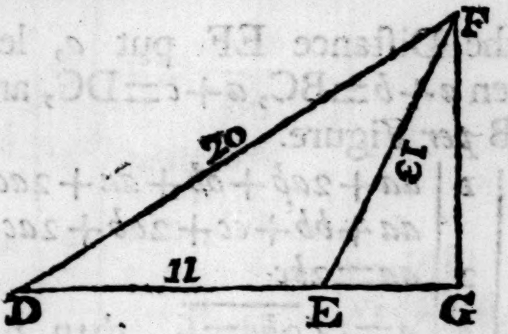
34. *In the Triangle ABC, the ſeveral Sides AB*$=13$, *AC*$=14$, *BC*$=15$ *are given; and the Perpendicular BD being drawn. Required the Segement of the Baſis AD, DC.*



Put $b=14$, $c=15$, $d=13$, AD$=a$, then DC$=b-a$, and let BD$=e$.

	1	$ee=dd-aa$ per E 47, 1
	2	$ee+aa-2ab+bb=cc$ D3
1, 2,	3	$dd-aa+aa-2ab+bb=cc$
$3-aa$	4	$2ab+cc=dd+bb$
$4-cc\div 2b$	5	$a=\frac{dd+bb-cc}{2b}=5.$
		then $14-5=9$ the Part DC.

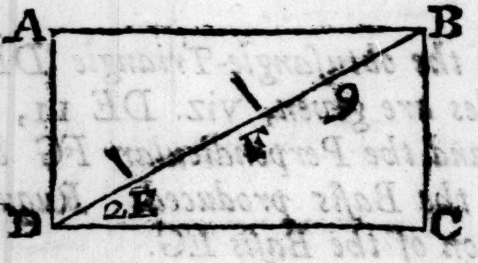
35. *In the obtuſangle-Triangle DEF, the ſeveral Sides are given,* viz. *DE* 11, *EF* 13, *DF* 20; *and the Perpendicular FG being let fall upon the Baſis produced. Ruquired the Prolongation of the Baſis EG.*



Let $b=11$, $c=13$, $d=20$, and put $a=$ EG and $e=$GF.

Then	1	$aa+ee=cc$	*per Fig.*
	2	$\left.\begin{matrix}aa+2ab\\+bb+ee\end{matrix}\right\}=dd$	
$2-1$	3	$2ab+bb=dd-cc$	
$3-bb\div 2b$	4	$a=\frac{dd-cc-bb}{2b}=5.$	

36. *In the Rectangle ABCD, is given the Difference between the Length AB and the Diogonal BD, that is DE*$=2$; *and likewiſe the Difference between the Breadth AD and the Diagonal BD, that is FB*$=9$. *Required the Sides of the Rectangle AB, AD?*



For the Diſtance EF put a, let $b=2$, $c=9$, then $a+b=$BC, $a+c=$DC, and $a+b+c=$DB *per* Figure.

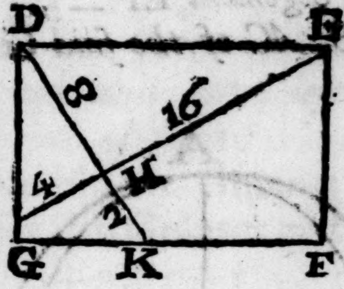
And | 1 | $aa+2ab+bb+aa+2ac+cc=aa+bb+cc+2ab+2ac+2bc$

hence | 2 | $aa=2bc$

2 *uu* | 3 | $a=\sqrt{2bc}=6$ then $6+2=8$ $=$BC, and $6+9=15=$DC.

37. *In a Rectangle DEFG, the right Line DK is drawn from the Angle D to the oppoſite Side, cutting the Diagonal EG at right Angles in H: And there is given the Segement HK*$=2$, *and HE*$=16$. *Required the Sides of the Rectangle?*

Put

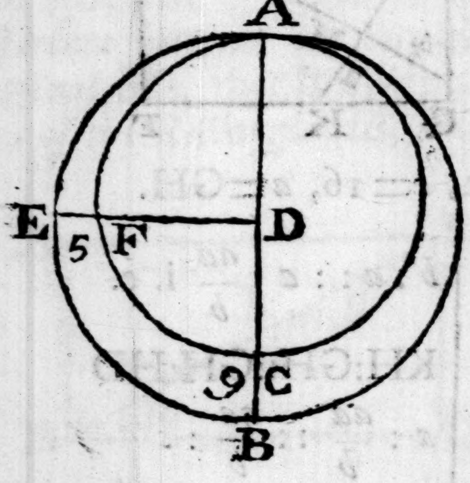


Put $b=2$, $c=16$, $a=$GH.

Then	1	$b : a :: a : \frac{aa}{b}$ i. e. KH:GH::GH:HD
∵	2	$a : \frac{aa}{b} :: \frac{aa}{b} : c$
	3	$ac = \frac{aaaa}{bb}$
$3 \times bb$ $\div a$	4	$aaa = bbc$
4 uu	5	$a = \sqrt[3]{bbc} = 4$ $2 : 4 :: 4 : 8 =$ HD *per* E 47, 1 DG $= 8.94426$, and DE $= 17.88854$.

38. *Let there be a Circle, whoſe Diameter is AB, with another leſs Circle whoſe Diameter which is AC, touches within, in A: And from the Center of the greater Circle D, draw the Radius DE at right Angles to AB, cutting the Periphery of the leſſer Circle in F. Now there is given BC (the Difference of the Diameters)* $=9$,

$=9$, with the Segement $EF=5$. Required the Diameters AB, AC of the ſaid Circles?



For the Radius of the greater Circle put a, $b=9$, $c=5$, then $DF=a-c$, and $DC=a-b$. Now *per* E 13, 6 DF is a main Proportional between DC and AD.

Ergo	1	$a-b : a-c :: a-c : a$
∵	2	$aa-ab=aa-2ac+cc$
$2-aa+2ac$	3	$2ac-ab=cc$
$3\div 2c-b$	4	$a=\dfrac{cc}{2c-b}=25$ which being doubled, gives the greater Diameter 50, and $50-9=41$ the leſſer.

39. *Two Companions have got a Parcel of Guineas; ſays A to B, if you will give me one of*

of your Guineas I ſhall have as many as you ſhall have left. Nay, replies B, if you will give me one of your Guineas, I ſhall have twice as many as you will have left. How many Guineas had each of them?

For the Number of Guineas which A had put a, and for B's Guineas put e.

Then	1	$a+1=e-1$	*per Queſtion.*
	2	$2a-2=e+1$	
$2-1$	3	$a-3=2$	
$3+3$	4	$a=5$ conſequently $e=7$.	

40. *A certain Perſon bought two Horſes, with the Trappings, which coſt* 100 *Pounds; which Trappings if laid on the firſt Horſe A, both the Horſes will be of equal Value: But if the ſaid Trappings be laid on the other Horſe, he will be double the Value of the firſt. How much did the ſaid Horſes coſt.*

Let $b=100$, and for the Value of the 2d Horſe and Trappings put a, then $b-a$ will be the firſt.

Then	1	$a=2b-2a$ *per Queſtion.*
$1+2a$	2	$3a=2b$
$2\div3$	3	$a=\frac{2b}{3}=66\frac{2}{3}$
and		$100-66\frac{2}{3}=33\frac{1}{3}$ the Value of the firſt Horſe, and $50-33\frac{1}{3}=16\frac{2}{3}$ the Price of the Trappings.

41. *A Vintner has two Sorts of Wine,* viz. *A and B; which if mix'd in equal Parts, a Flaggon of mix'd will coſt* 15 *Pence; but if they be mix'd in a* ſeſqui-alter *Proportion, as if you ſhould take* 2 *Flaggons of A as often as you take* 3 *of B, a Flaggon will coſt* 14 *Pence. Required the Price of each Wine ſingly?*

Let $b=15$, $c=14$, for the Value of the Flaggon A put a, for that of B put e.

Then	1	$a+e=2b$	*per Queſt.*
and	2	$2a+3e=5c$	
1×3	3	$3a+3e=6b$	
$3-2$	4	$a=6b-5c=20$, and $2b-20=10$.	

42. *A Son aſked his Father how old he was; his Father anſwered him thus: If you take away* 5 *from my Years, and divide the Remainder by* 8, *the Quotient will be* $\frac{1}{3}$ *of your Age: But if you add* 2 *to your Age, and multiply the whole by* 3, *and then ſubſtract* 7 *from the Product, you will have the Number of the Years of my Age. What was the Age of the Father and Son?*

Let $a=$ the Father's Age, and $e=$ the Son's.

Then	1	$\frac{a-5}{8}=\frac{1}{3}e$ *per Queſtion.*
1×24	2	$3a-15=8e$
	3	$3e+6-7=a$ *per Queſtion.*
3×3	4	$9e+18-21=3a$
$4-2$	5	$e=18$, and $3e+6-7=53=a$.

43. *To find out 2 Numbers, to the Sum whereof if you add 6, the whole ſhall be double the greater; and if you ſubſtract 2 from their Difference, the Remainder will be half of the leaſt.*

Let $b=6$, $c=2$, $a=$ the greater Number, and $e=$ the leſs.

Then	1	$a+e+b=2a$	*per Queſt.*
	2	$a+e-c=\frac{1}{2}e$	
2, × 2	3	$2a-2e-2c=e$	
$3+2e+2c$	4	$2a=3e+2c$	
$1-a$	5	$a=e+b$	
4, 5,	6	$2e+2b=3e+2c$	
6,	7	$e=2b-2c=8$, and, from the 5th Step $a=14$.	

44. *To find two Numbers, the Product whereof is* 240, *and the Triple of the greater divided by the leſs is* 5.

Let $b=240$, $c=5$, $a=$ the greater, $e=$ the leſs.

Then	1	$ae=b$	*per Queſtion.*
	2	$\frac{3a}{e}=c$	
$2\times e$	3	$3a=ec$	
$3\div c$	4	$e=\frac{3a}{c}$	
1, 4,	5	$\frac{3aa}{c}=b$	
5 uu	6	$a=\sqrt[2]{\frac{bc}{3}}=20$, and $240\div20=12=e$.	

45. *Two Men have a Mind to purchaſe a Houſe rated at* 1200 *l. ſays A to B, if you give me* $\frac{2}{3}$ *of your Money, I can purchaſe the Houſe alone; but ſays B to A if you will give me* $\frac{3}{4}$ *of yours, I ſhall be able to purchaſe the Houſe. How much Money had each of them?*

Let $b=1200$, for A's Money put a, and for B's put e.

Then	1	$a+\frac{2}{3}e=b$	*per Queſtion.*
	2	$e+\frac{3}{4}a=b$	
1 × 3	3	$3a+2e=3b$	
2 × 4	4	$4e+3a=4b$	
3 × 2	5	$6a+4e=6b$	
5 − 4	6	$3a=2b$	
6 ÷ 3	7	$a=\frac{2b}{3}=800$, conſequently $e=600$.	

46. *Some young Men and Maids had a Reckoning of* 37 *Crowns to pay for a Treat, and this was their Conditions, that every young Man ſhould pay* 3 *Crowns, and every Maid* 2. *Now, if there had been as many young Men as there were Maids, obſerving the ſame Conditions, the Reckoning would have come to* 4 *Crowns leſs than it did. How many young Men and Maids were there?*

Let $b=37$, $c=33$, $a=$ the Number of Men, $e=$ the Maids.

Then

Then	1	$3a+2e=b$	per *Queſtion.*
	2	$3e+2a=c$	
1 × 2	3	$6a+4e=2b$	
2 × 3	4	$6a+9e=3c$	
4 − 3	5	$5e=3c-2b$	
5 ÷ 5	6	$e=3c-2b=5$, and from the firſt Step a will be found $=9$.	

47. *A General who had fought a Battle, upon reviewing his Army, whoſe Foot was thrice the Number of his Horſe, finds that before the Battle* $\frac{1}{12}-120$ *of his Foot had deſerted, and of his Horſe* $\frac{1}{20}+120$, *beſides* $\frac{1}{4}$ *of his whole Army were ſent into Garriſons (reckoning the Sick and Wounded) and* $\frac{3}{8}$ *of his Army remain'd; the reſt, who were wanting, being either ſlain or taken Priſoners; now if you add* 3000 *to the Number of the Slain, the Sum will be equal to half the Foot he had at the Beginning. What were the Numbers of each?*

Let $b=3000$, $a=$ the Number of Horſe, then $3a=$ the Foot, and $4a=$ the whole Army; now the Number of the Horſe and Foot deſerted, and the 4th Part ſent into Garriſon, together with $\frac{3}{8}$ that remains, will be $\frac{3a}{12}+\frac{a}{20}+a+\frac{12a}{8}$ the Sum of which will be $\frac{14a}{5}$ which taken from $4a$ the whole Army, there remains $\frac{6a}{5}$ equal the Number of Slain.

Then	1	$\frac{6a}{5}+b=\frac{3a}{2}$
1×10	2	$12a+10b=15a$
$2-12\div 3$	3	$a=\frac{10b}{3}=10000$, and $3a=30000=$ the Foot.

48. *To divide* 100 *twice into two Parts, ſo that the major Part of the firſt Diviſion may be triple the minor Part of the ſecond Diviſion; and the major Part of the ſecond may be double the minor Part of the firſt.*

Let $b=100$, and $a=$ the major Part of the Diviſion, then $b-a=$ the minor, $\frac{a}{3}=$ the minor Part of the ſecond, $2b-2a=$ the major Part of the ſecond *per Queſtion*.

Then	1	$2b-2a+\frac{a}{3}=b$
1×3	2	$6b-6a+a=3b$
$2\div 5$	3	$a=\frac{3b}{5}=60$, from whence the other Parts will be found to be $40=$ the leſſer Part of the firſt, 80 the greater, and 20 the leſſer Part of the ſecond.

49. *To divide* 30 *twice into two Parts, ſo that the major Part of the firſt Diviſion, with the minor of the ſecond may be* 33; *and the Sum of the minor Parts ſubſtracted from the Sum of the major, may leave* 14 *remaining.*

Let $b=30$, $c=33$, $d=14$, and $a=$ the major Part of the firſt Diviſion, then $b-a=$ the leſſer, $c-a=$ the leſſer Part of the 2d, and $b-c+a=$ the major Part of the 2d, the two minor Parts being ſubſtracted from the Major, the Remainder will be $4a-2c=d$.

Hence	1	$4a=d+2c$
$1\div4$	2	$a=\frac{d+2c}{4}=20$, from whence the other Parts will be found to be 10, 17, and 13.

50. *A Man, his Wife, and his Son's Ages make up* 96 *Years, ſo that the Huſband's and Son's Years together make the Wife's* + 15; *but the Wife's and the Son's make the Huſband's* + 2. *What was the Age of each?*

Let $b=96$, $c=15$, $d=2$, and for the Huſband's Age put a, for the Wife's e, and for the Son's y.

Then	1	$a+e+y=b$	
	2	$a+y=e+c$	per *Queſtion.*
	3	$e+y=a+d$	
$1-3$	4	$a=b-a-d$	
$4+a\div2$	5	$a=\frac{b-d}{2}=47$	
$1-2$	6	$e=b-e-c$	
$6+e\div2$	7	$e=\frac{b-c}{2}=40\frac{1}{2}$,	
$1-a-e$	8	$y=b-a-e=8\frac{1}{2}$.	

51. *Three Merchants from three different Fairs meet together at an Inn, where they reckon up their Gains, and find them the Sum of 780 Crowns. Moreover, if you add the Gain of the firſt and ſecond, and ſubſtract the Gain of the third from the Sum, there remains the Gain of the firſt + 82 Crowns; but if you add the Gain of the ſecond and third, and from the Sum ſubſtracted the Gain of the firſt, there remains the Gain of the third — 43 Crowns. What was the Gain of each?*

Let $b=780$, $c=82$, $d=43$, and let $a=$ the firſt, $e=$ the ſecond, and $y=$ the third Merchant's Gain.

Then	1	$a+e+y=b$	*per Queſt.*
	2	$a+e-y=a+c$	
	3	$e+y-a=y-d$	
$1-2$	4	$2y=b-a-c$	
$1-3$	5	$2a=b-y+d$	
5×2	6	$4a=2b-2y+2d$	
$6+2y$	7	$4a+2y=2b+2d$	
$7-4$	8	$4a=b+a+2d+c$	
$8-a\div 3$	9	$a=\frac{b+c+2d}{3}=316,$	
$4\div 2$	10	$y=\frac{b-a-c}{2}=191,$	
$1-a-y$	11	$e=b-a-y=273.$	

52. *Three Perſons, A, B, C, owe a certain Sum of Money, ſo that A and B together owe* 210 *Crowns; B and C* 290, *and A and C* 400. *What did each of them owe?*

Let $b=210$, $c=290$, $d=40$, for the Sum that A ow'd put a, what B ow'd put e, and for what C ow'd put y.

Then	1	$a+e=b$
	2	$e+y=c$ *per Queſtion.*
	3	$a+y=d$
$1+3$	4	$2a+e+y=b+d$
$4-2\div2$	5	$a=\frac{b+d-c}{2}=160$,
		and from the 1ſt Step $e=50$,
		and from the 3d, $y=240$.

53. *To find three Numbers, ſo that the firſt and half the Remainder, the ſecond and* $\frac{1}{3}$ *of the Remainder, and the third and* $\frac{1}{4}$ *of the Remainder, may always make* 34.

Let $b=34$, $a=$ the firſt, $e=$ the ſecond, and $y=$ the third Number.

Then	1	$a+\frac{e+y}{2}=b$	*per Queſt.*
	2	$e+\frac{a+y}{3}=b$	
	3	$y+\frac{a+e}{4}=b$	
1 × 2	4	$2a+e+y=2b$	
2 × 3	5	$3e+a+y=3b$	
3 × 4	6	$4y+a+e=4b$	
5+6	7	$4e+2a+5y=7b$	
7 — 4	8	$3e+4y=5b$	
6 — 5	9	$3y-2e=b$	
8 × 2	10	$6e+8y=10b$	
9 × 3	11	$9y-6e=3b$	
10+11	12	$17y=13b$	
12÷17	13	$y=\frac{13b}{17}=26$	
$8-4y\div 3$	14	$e=\frac{5b-4y}{3}=22$	
$4-e-y\div 2$	15	$a=\frac{2b-e-y}{2}=10.$	

54. *Let a Square be divided into 9 ſmall Squares: We are to find and diſpoſe the Numbers through the ſeveral ſmall Areas, ſo that the*

the Sum of every three, taken either laterally or diagonally may be always 15.

In all Queſtions of this Nature, if the Number of given Squares are even, they are to be ſolv'd in a mechanical Way, but where they are odd, as in this Example the middle Number may be found analytically, but the reſt mechanically after the following Method, ſupply the given Squares with Letters of the Alphabet.

	1	$a+e+i=15$ } *per Queſtion.*
	2	$c+e+g=15$
	3	$b+e+h=15$
$1+2+3$	4	$a+3e+i+c+g+b+h=45$
	5	$a+b+c=15$ } *per Queſtion.*
	6	$g+h+i=15$
$5+6$	7	$a+b+c+g+h+i=30$
$4-7$	8	$3e=15$
$8\div3$	9	$e=\frac{15}{3}=5$ which being placed in the middle Square, the other Squares will be eaſily ſupply'd with Numbers, as in the annex'd Diagram.

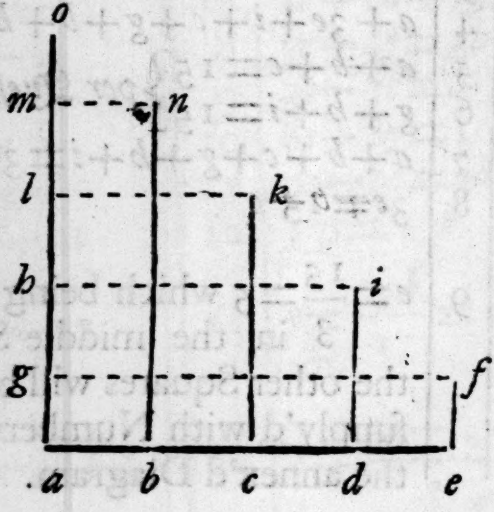
a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

55.

55. *(Theorem) Let any Numbers whatſoever be given, if you ſubſtract every leſs Number from that which is the next greateſt: I ſay, that the Sum of thoſe Differences is equal to the Difference of the greateſt and leaſt Numbers.*

DEMONSTRATION.

'Tis evident from Fig. below that $ao-nb=l=om$, $nb-kc=lm$, $kc-id=lh$, $id-fe=hg$, now 'tis alſo evident that the Sum of thoſe Differences, *viz.* $om+ml+lh+hg=og=ao$ W. W. D.



56. *To find a Number, which being multiplied by* 6, *and the Product ſubſtracted from the Square of the Number to be found, the Remainder will be* 280.

Let $b=6$, $c=280$, and let $a+\frac{1}{2}b=$ the Number ſought.

Then

Then	1	$\left.\begin{matrix} aa+ba+\frac{1}{4}bb \\ -ba-\frac{1}{2}bb \end{matrix}\right\}=c$ *per Queſt.*
1	2	$aa+\frac{1}{4}bb=c$
2,	3	$a=\sqrt[2]{c+\frac{1}{4}bb}=17$ $17+\frac{1}{2}b=20$ the N°. ſought.

57. *To find a Number which being multiplied by* 8, *and the Product added to the Square of the Number to be found, the Sum will be* 660.

Let $b=8$, $c=660$, and let $a-\frac{1}{2}b=$ the Number ſought.

Then	1	$ba-\frac{1}{2}bb$
	2	$aa-ab+\frac{1}{4}bb$
$1+2$	3	$aa-\frac{1}{4}bb=c$ *per Queſtion.*
$3+\frac{1}{4}b6$	4	$a=\sqrt[2]{c+\frac{1}{4}bb}=26$
then	5	$a-\frac{1}{2}b=22$ the Num. ſought.

58. *To divide* 140 *into two Parts, ſo that the Product of thoſe Parts may* = *the Square of* 56, *that is* 3136.

Let $b=140$, $c=3136$, and let $a=$ one of the Parts, and $e=$ the other.

Then

Then	1 2	$\left.\begin{array}{l} a+e=b \\ ae=c \end{array}\right\}$ *per Queſtion.*
1 ©	3	$aa+2ae+ee=bb$
2 × 4	4	$4ae=4c$
3—4	5	$aa-2ae+ee=bb-4c$
5 *uu*	6	$a-e=\sqrt[2]{bb-4c}$
1+6÷2	7	$a=\frac{\sqrt[2]{bb-4c}+b}{2}=112,$
1—*a*	8	$e=b-a=28.$

59. *Let* 969 *Soldiers be drawn up into an oblong Battle, ſo that the Difference of the greater and leſs Sides is* 40. *Required the Number of the Soldiers in each Rank, in Length and Breadth.*

Let $b=969$, $c=40$, $a=$ the greater, and $e=$ the leſſer Sides.

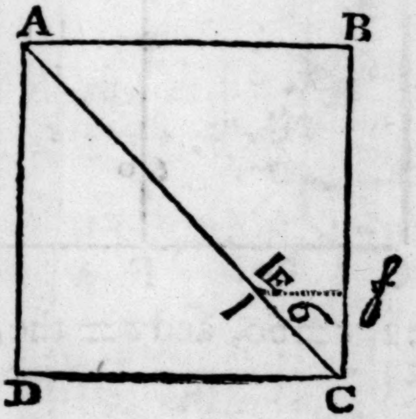
Then	1	$ae=b$
	2	$a-e=c$
2 ©	3	$aa-2ae+ee=cc$
1 × 4	4	$4ae=4b$
3+4	5	$aa+2ae+ee=cc+4b$
5 *uu*	6	$a+e=\sqrt[2]{cc+4b}$
2+6÷2	7	$a=\frac{\sqrt[2]{cc+4b}+c}{2}=57$
and	8	$a-c=e=27.$

60. *Again, let* 480 *Soldiers be drawn up into an oblong Battle, ſo that the Sum of the greater and leſs Sides is* 52. *Required the Number of the Soldiers of each Rank in Length and Breadth?*

Let $b=480$, $c=52$, $a=$ the greater, and $e=$ the leſſer Side.

Then	1	$ae=b$ } *per Queſtion.*
	2	$a+e=c$
2 ⊚	3	$aa+2ae+ee=cc$
1 × 4	4	$4ae=4b$
3—4	5	$aa-2ae+ee=cc-4b$
5 uu	6	$a-e=\sqrt[2]{cc-4b}$
2+6÷2	7	$a=\frac{\sqrt[2]{cc-4b}+c}{2}=40$
2—*a*	8	$e=c-a=12$.

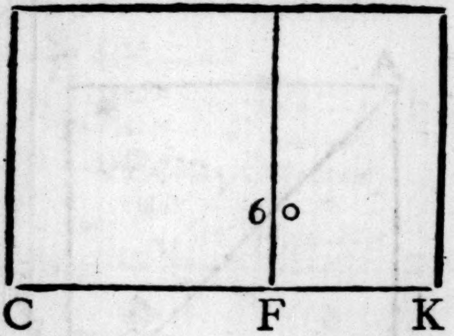
61. *In the Square ABCD, is given the Difference of the Diagonal and the Side, that is EC=6. Required the Side of the Square?*



Let $b=6$, draw the Line E f, now 'tis plain, that the Square of E f + the Square of f C = the Square of E C, and E f = f C, therefore $\frac{\sqrt[2]{bb}}{2}$ = E f, or F C, which make = $c=4.2426289$, and for the Side put a.

Then	1	$b : c :: a+b : a$ *per* E 6, 6
∵	2	$ac+cb=ba$
$2-ac$	3	$ba-ac=cb$
$3 \div b-c$	4	$a=\frac{cb}{b-c}=14.485=$ the Side of the Square.

62. *The Rectangle EK is added to the Square DF (being of the ſame Height) whoſe Breadth EL is given* = 2, *and alſo the Area of the whole compound Rectangle DK,* = 60. *required the Side of the Square?*



Let $b=2$, $c=60$, and $a=$ the Side of the Square.

Then

Then	1	$aa+ab=c$ *per Queſtion.*
$\square\, c$	2	$aa+2b+\frac{1}{4}bb=c+\frac{1}{4}bb$
2 *uu*	3	$a+\frac{1}{2}b=\sqrt{c+\frac{1}{4}bb}$
$3-\frac{1}{2}b$	4	$a=\sqrt{c+\frac{1}{4}bb}-\frac{1}{2}b=6.8102.$

63. *A Man buys ſome Ells of Cloth for* 70 *Crowns; and finds that if he had* 4 *Ells more, he had then bought every Ell* 2 *Crowns cheaper. How many Ells did he buy?*

Let $b=70$, $c=4$, $d=2$, $a=$ the Number of Ells, and $e=$ the Crowns each Ell coſt.

Then	1	$ae=b$ *per Queſtion.*
and	2	$\left.\begin{array}{l}a+c\times e-d=ae\\+ec-ad-dc=b\end{array}\right\}$ *per Queſt.*
1, 2,	3	$ec=ad+dc$
$3\div c$	4	$e=\frac{ad+dc}{c}$
1, 4,	5	$\frac{aad+adc}{c}=b$
$5\times c\div d$	6	$aa+ac=\frac{cd}{d}$
6, $\square\, c$	7	$aa+ac+\frac{1}{4}cc=\frac{cb}{d}+\frac{1}{4}cc$
7, *uu*	8	$a+\frac{1}{2}c=\sqrt[2]{\frac{cb}{d}+\frac{1}{4}cc}$
$8-\frac{1}{2}c$	9	$a=\sqrt{\frac{cb}{d}+\frac{1}{4}cc}-\frac{1}{2}c=10\ \&\ e=7.$

64. *A Set of boon Companions dining at an Inn, the Reckoning in all came to* 175 *Shillings: But, before the Bill was paid off, two of them ſlunk away, and then the Club of thoſe that remain'd came to* 10 *Shillings a Man more. How many were there in Company?*

Let $b=175$, $c=2$, $d=10$, $a=$ the Number of Men, and $e=$ the Number of Shillings each ſhould have ſpent if they had all ſtay'd.

Then	1	$ae=b$
and	2	$ae+ad-ce-dc=b$ *per Queſt.*
1, 2,	3	$a=\frac{ce+dc}{d}$
1, 3,	4	$\frac{eec+dec}{d}=b$
$4\times d\div c$	5	$ee+de=\frac{bd}{c}$
□ c	6	$ee+de+\frac{1}{4}dd=\frac{bd}{c}+\frac{1}{4}dd$
6 *uu*	7	$e+\frac{1}{2}d=\sqrt{\frac{bd}{c}+\frac{1}{4}dd}$
$7-\frac{1}{2}d$	8	$e=\sqrt{\frac{bd}{c}+\frac{1}{4}dd}-\frac{1}{2}d=25$
1, 8	9	$a=\frac{b}{e}=7.$

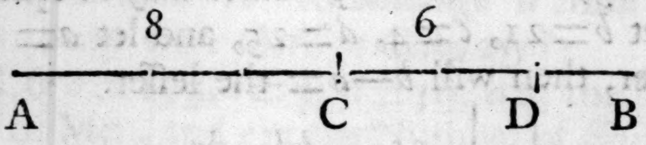
65. *To divide the Number* 21 *into* 2 *Parts, ſo that if the greater be divided by the leſs; and again the leſs by the greater, and then the firſt Quotient being multiply'd by* 4, *and the latter by* 25, *the Numbers produced may be equal.*

Let $b=21$, $c=4$, $d=25$, and let $a=$ the greater, then will $b-a=$ the leſſer.

Then	1	$\frac{ac}{b-a}=\frac{bd-da}{a}$
$1\times b-a\times a$	2	$aac=bbd-2abd+aad$
Signs Ch	3	$aad-aac-2abd=-bbd$
		let $d-c=f$
then	4	$faa-2abd=-bbd$
$4\div f$, □C	5	$aa-\frac{2abd}{f}+\frac{bbdd}{ff}=\frac{bbd}{f}+\frac{bbdd}{ff}$
5, *uu*	6	$aa-\frac{bd}{f}=\sqrt[2]{\frac{bbdd}{ff}-\frac{bbd}{f}}$
$6+\frac{bd}{f}$	7	$a=-\sqrt{\frac{bbd}{f}+\frac{bbdd}{ff}}+\frac{bd}{5}=25$
then	8	$25-10=15$
and	9	$b-15=6$ the leſſer.

66. *Let the Line AB be divided in C, ſo that AC may be* 8 *and CB* 6: *We are to divide the ſame Line AB in D, ſo that the*

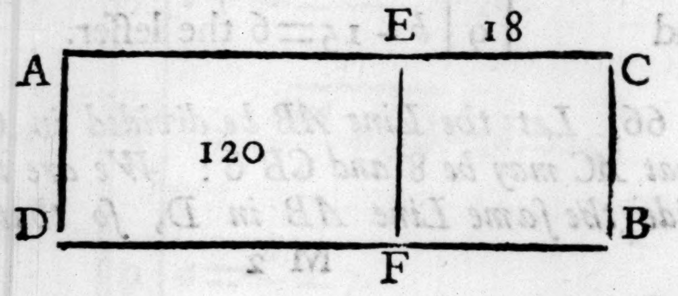
Rectangle under AD and DC may be equal to the Rectangle under AC and CB, or to the Product from 8 *and* 6, *which is* 48. *Required the Segement CD?*



Let $b=8$, $c=48$, and put $a=$CD.

Then	1	$a+b=\text{AD}$
and	2	$aa+ab=c$ *per Queſtion.*
2, □ C	3	$aa+ab+\frac{1}{4}bb=c+\frac{1}{4}bb$
3 *uu*	4	$a+\frac{1}{2}b=\sqrt[2]{c+\frac{1}{4}bb}$
$4-\frac{1}{2}b$	5	$a=\sqrt{c+\frac{1}{4}bb}-\frac{1}{2}b=4.$

67. *Let there be a Rectangular Garden, ABCD, the Length of which AB is thrice the Breadth AD; and reckoning* 18 *Perches from B towards A, that is BE, and reckoning EF parallel to AD, let the Area of the remaining Rectangle ED be given* = 120 *ſquare Perches. What was the Length and Breadth of the ſaid Garden?*



Let $b=18$, $c=120$, and put $a=$ AE, then will $a+b=$ the whole Length, and $\frac{a+b}{3}=$ the Breadth.

Then	1	$\frac{aa+2ab+bb}{3}=c+\frac{ab+bb}{3}$ *p.Q.*
$1\times3-bb$	2	$aa+ab=3c$
$2\ \square\ c$	3	$aa+ab+\frac{1}{4}bb=3c+\frac{1}{4}bb$
$3\ uu$	4	$a+\frac{1}{2}b=\sqrt{3c+\frac{1}{4}bb}$
$4-\frac{1}{2}b$	5	$a=\sqrt{3c+\frac{1}{4}bb}-\frac{1}{2}b=12$
	6	$a=12+18=30$ the Length
and	7	$\frac{30}{3}=10$ the Breadth.

68. *Let* 600 *Soldiers be diſpoſed into an oblong Battle: Which the Colonel willing to make broader, finds that if he takes away* 10 *Ranks from the Length, he ſhall augment the Breadth with two Ranks. What was the Number of his Soldiers through every Rank in Length and Breadth?*

Let $b=600$, $c=10$, $d=2$, $a=$ the Length, and $e=$ the Breadth, $a-c=$ the diminiſhed Length, and $e+b=$ the augmented Breadth.

Then

Then	1	$ae=b$
and	2	$ae+ad-ec-dc=b$ *per Queſt.*
$2+dc+ce$	3	$ad=dc+ce=b$
$3\div d$	4	$a=\frac{dc+ce}{d}$
1, 4,	5	$\frac{cee+dce}{d}=b$
$5\times d\div c$	6	$ee+de=\frac{bd}{c}$
$6 \square c$	7	$ee+de+\frac{1}{4}dd=\frac{bd}{c}+\frac{1}{4}dd$
$7\,uu-\frac{1}{2}d$	8	$e=\sqrt{\frac{bd}{c}+\frac{1}{4}dd}-\frac{1}{2}d=10$
1,	9	$a=\frac{b}{10}=60.$

69. *A Man buys a Horſe, which he ſells again for 56 Crowns, and gains as many Crowns in* 100 *as the Horſe coſt him. How much did he give for the Horſe?*

Let $b=56$, $c=100$, $a=$ the firſt Price, then will $b-a=$ his Gain by the Sale.

And

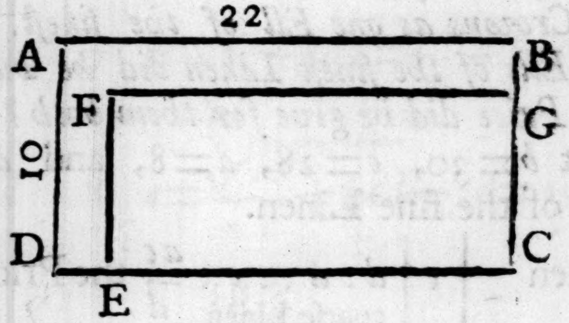
And	1	$c : a :: a : b-a$
∵	2	$aa=bc-ac$
$2+ac$	3	$aa+ac=bc$
3 □ c	4	$aa+ac+\frac{1}{4}cc=bc+\frac{1}{4}cc$
4 $uu-\frac{1}{2}c$	5	$a=\sqrt{bc+\frac{1}{4}cc}-\frac{1}{2}c=40$,
and	6	$b-a=16$ his Gain by the Sale for 100 : 40 :: 40 : 16.

70. *A certain Linen-Draper buys two ſorts of Linen for* 30 *Crowns, one finer, the other coarſer: An Ell of the fineſt coſt as many Crowns as he had Ells: And alſo* 28 *Ells of the coarſeſt at ſuch a Price, that* 8 *Ells coſt as many Crowns as one Ell of the fineſt. How many Ells of the fineſt Linen did he buy, and what Price did he give for them both?*

Let $b=30$, $c=28$, $d=8$, and $a=$ the Price of the fine Linen.

Then	1	$d : a :: c : \frac{ac}{d}$ the Price of the coarſe Linen.
and	2	$aa+\frac{ac}{d}=b$ per *Queſtion.*
2 □ c	3	$aa+\frac{ac}{d}+\frac{\frac{1}{4}cc}{dd}=b+\frac{\frac{1}{4}cc}{dd}$
3 uu	4	$a+\frac{\frac{1}{2}c}{d}=\sqrt{b+\frac{\frac{1}{4}cc}{dd}}$
$4-\frac{\frac{1}{2}c}{d}$	5	$a=\sqrt{b+\frac{\frac{1}{4}cc}{dd}}-\frac{\frac{1}{2}c}{d}=4$
and	6	$4\div 8=\frac{1}{2}$ Crown the Price of the coarſer.

71. *In a certain Rectangular Garden, the Length of which AB, is* 22 *Perches, and the Breadth AD is* 10, *the Walk DG is to be made, in a Situation parallel to the Sides of the Figure, ſo that the Area of the ſaid Walk or Gnomon DG may be equal to the remaining Rectangle FC, or that the Gnomon DG may be half of the whole Figure ABCD propos'd. Required the Breadth of the ſaid Gnomon DE or BG?*



Let $b = 22$, $c = 10$, $d = \frac{bc}{2} = 110$, $a =$ the Length of the Garden when the Walk is taken off it, and $e =$ the Breadth thereof.

Then

Then	1	$ae=d$
and	2	$b-a=c-e$ *per Queſtion*, being each the Breadth of the Walk.
2+a+e	3	$c+a=b+e$
3—b	4	$e=c+a-b$
1, 4	5	$aa+ac-ab=d$
	6	$aa-12a=d$
6 □ c	7	$aa=12a+36=d+36$
1 *uu* + 6	8	$a=\sqrt[2]{d+36}+6=18.083$
and	9	$b-a=39169541$ the Breadth of the Walk.

72. *Of three proportional Numbers there is the middle Term given,* $=12$, *and the Difference of the Extremes* $=10$. *Required the Extremes?*

Let $b=12$, $c=10$, and $a=$ the leſſer Extreme.

Then	1	$a:b::b:a+c$
∵	2	$aa+ac=bb$
2 □ c	3	$aa+ac+\frac{1}{4}cc=bb+\frac{1}{4}cc$
3 *uu*	4	$a+\frac{1}{2}c=\sqrt{bb+\frac{1}{4}cc}$
4—$\frac{1}{2}c$	5	$a=\sqrt{bb+\frac{1}{4}cc}-\frac{1}{2}c=8$
and	6	$8+10=18$ the greater.

73. *Of three proportional Numbers there is given the Sum of the firſt and ſecond,* $=10$, *and the Difference of the ſecond and third* $=24$. *Required the ſeveral Numbers?*

Let $b=10$, $c=24$, $a=$ the middle Term ſought, then will $b-a=$ the firſt, and $a+c=$ the third.

Then	1	$b-a : a :: a : a+c$
∴	2	$aa=bc+ab-ca-aa$
$2+aa$ &c.	3	$2aa+ca-ba=bc=240$
$2\div 2$	4	$aa+7a=120$
4 □ c	5	$aa+7a+\frac{49a}{4}=120+\frac{49}{4}$
5 *uu*	6	$a+\frac{7}{2}=\sqrt{120+\frac{49}{4}}$
$6-\frac{7}{2}$	7	$a=\sqrt{120+\frac{49}{4}}-\frac{7}{2}=8.$

74. *Of four proportional Numbers there is given the third* $=12$, *alſo the Sum of the firſt and ſecond* $=8$; *beſides the ſecond Number being ſubſtracted from its Square, the Remainder is to be the fourth. Required the ſaid Numbers?*

Let $b=12$, $c=8$, and $a=$ the ſecond Term, then the firſt will be $c-a$, and the 4th will be $aa-a$.

Then

Then	1	$c-a : a :: b : aa-a$
∵	2	$ba = caa + aa - aaa - ca$
$2+a^3$&c.	3	$aaa - caa - aa = -ba - ca$
$3 \div a$	4	$aa - ca - a = -b - c$
inNumb.	5	$aa - 9a = -20$
$5 \square c$	6	$\left.\begin{array}{l} aa-9a+20.25 \\ =-20+20.25 \end{array}\right\} =25$
6 *uu*	7	$a-4.5=\sqrt{.25}$
7+4.5	8	$a=\sqrt{.25}+4.5=5.$
then	9	8−5=3 the firſt,
and	10	25−5=20 the fourth.

75. *Of four Numbers in continued Proportion there is given the Sum of the Means* $=24$, *and likewiſe the Sum of the Extremes* $=56$, *Required the ſaid Numbers (ſuppoſing that the firſt is the leaſt of all?)*

Let $2b=24$, $c=56$, and $2a=$ the Difference of the 2d and 3d Terms, then will $b-a=$ the 2d, and $b+a=$ the 3d.

∵	1	$\left.\begin{matrix} b+a : b-a :: b-a : \\ \dfrac{aa-2ab+bb}{b+a} \end{matrix}\right\} = \text{the 1st}$
and ∵	2	$\left.\begin{matrix} b-a : b+a :: b+a : \\ \dfrac{aa+2ab+bb}{b-a} \end{matrix}\right\} = \text{the 4th}$
then	3	$\left.\begin{matrix} \dfrac{aa+2ba+bb}{b-a} \\ +\dfrac{aa-2ab+bb}{b+a} \end{matrix}\right\} = c \text{ per } Q.$
$3 \times b+a$	4	$\dfrac{aaa+3aab+3abb+bbb}{b-a} + aa-2ab+bb = bc+ca$
$4 \times b-a$	5	$\left.\begin{matrix} aaa+3aab \\ +3abb+bbb \\ -aaa+3aab \\ -3abb+bbb \end{matrix}\right\} = bbc-caa$
5,	6	$6aab+caa = bbc-2bbb$
$6 \div 6b+c$	7	$aa = \dfrac{bbc-2bbb}{6b+c}$
7 *uu*—	8	$a = \dfrac{\sqrt[2]{bbc-2bbb}}{6b+c} = 6$
		and $b+a=18$, $b-a=6=$ the 2d and 3d Terms.
then	9	$6:18::18:54=$ the 4th Term.
and	10	$56-54=2=$ the firſt Term.

76. *Two Country-women A and B, carry* 100 *Eggs together to Market; and in the Sale of them, one took as much Money as the other; but A (who had the largeſt and conſequently the beſt Eggs) ſays to B, had I carried as many Eggs as you, I ſhould have had* 18 *Pence for them; B replies, if I had brought as many Eggs as you, I ſhould have had but* 8 *Pence for them. How many Eggs had each?*

Let $b=100$, $c=18$, $d=8$, A's Eggs $=a$, then will B's $=b-a$, and let $e=$ the Numb. of A's *per* Penny, and $y=$ B's.

Then	1	$ae=by-ay$	
	2	$be-ae=c$	*per Queſtion.*
	3	$ay=d$	
$2\div b-a$	4	$e=\frac{c}{b-a}$	
$3\div a$	5	$y=\frac{d}{a}$	
1, 4, 5,	6	$\frac{bd}{a}-d=\frac{ac}{b-a}$	
$6\times a$	7	$bd-ad=\frac{aac}{b-a}$	
$7\times b-a$	8	$aac=bbd-2abd+aad$	
8,	9	$aac-aad+2abd=bbd$	
inNumb.	10	$10aa+1600a=80000$	
$10\div 10$	11	$aa+160a=8000$	
11 □ *c*	12	$aa+160a+6400=8000+6400$	
12 *uu*	13	$a+80=\sqrt{8000+6400}=120$	
13—80	14	$a=120-80=40=$ A's Eggs	
and	15	$100-40=60=$ B's Eggs.	

77. *Two Countrymen* A *and* B *ſell their Corn at different Prices:* A *ſells* 20 *Buſhels; and* B *received for one Buſhel as many Crowns as he ſold Buſhels:* A *perceives that if he had ſold as many Buſhels as* B *received Crowns, he ſhould then have received* 252 *Crowns; but both together received* 176 *Crowns. How many Buſhels did* B *ſell, and what Price had* A?

Let $b=20$, $c=252$, $d=176$, and $a=$ the Price of A's Corn *per* Buſhel, and let $e=$ B's Buſhels, then the Number of Crowns he received will be ee.

Then	1	$ab+ee=d$ } *per Queſtion.*
and	2	$aee=c$
$2\div a$	3	$ee=\frac{c}{a}$
1, 3	4	$ab+\frac{c}{a}=d$
$4\times a$	5	$aab+c=ad$
$5-ad-c$	6	$aab-ad=-c$
$6\div b$	7	$aa-\frac{ad}{b}=-\frac{c}{b}$
7 □ Ç	8	$aa-\frac{ad}{b}+\frac{1}{4}\frac{dd}{bb}=-\frac{c}{b}+\frac{1}{4}\frac{dd}{bb}$
$8uu+\frac{1}{2}\frac{d}{b}$	9	$a=\sqrt[2]{-\frac{c}{b}+\frac{1}{4}\frac{dd}{bb}}+\frac{1}{2}\frac{d}{b}=7$
		then $7\times20=140=$ A's Money, and $176-140=36=$ B's Money, the Square Root of which, *viz.* 6 is $=$ his Number of Buſhels.

78. *Two Merchants ſell* 21 *Ells of Cloth: The firſt ſells one Ell for as many Crowns as is*

$\frac{1}{5}$ of the Number of Ells that the ſecond had; and the ſecond ſells one Ell for as many Crowns as is $\frac{1}{3}$ of the Number of the Ells that the firſt had. The Sale being over, they had taken 48 *Crowns in all. How many Ells did each ſell, and at what Price.*

Let $b=21$, $c=48$, $a=$ the firſt Merchant's Ells, then will $b-a=$ the Second's, $\frac{a}{3}$ and $\frac{b-a}{5}=$ the Price each ſold his Cloth *per* Ell.

Then	1	$\frac{ab-aa}{5}+\frac{ab-aa}{3}=c$ *per Qu.*
1×15	2	$3ab-3aa+5ab-5aa=15c$
2,	3	$-8aa+8ab=15c$
Sign Ch.	4	$8aa-8ab=-15c$
$4\div 8$	5	$aa-ab=-\frac{15c}{8}$
$5\ \square\ c$	6	$aa-ab+\frac{1}{4}bb=-\frac{15c}{8}+\frac{1}{4}bb$
$6\ uu$	7	$a-\frac{1}{2}b=\sqrt{-\frac{15c}{8}+\frac{1}{4}bb}$
$7+\frac{1}{2}b$	8	$a=-\sqrt[2]{\frac{15c}{8}+\frac{1}{4}bb}+\frac{1}{2}b=15$
then	9	$21-15=6$ the Number of B's Ells, $\frac{a}{5}=3$ Crowns what the

2d Merchant ſold his Cloth for *per* Ell.

And $\frac{b-a}{3}=2$ what the firſt Merchant ſold his for *per* Ell.

79. *Two Merchants have a Parcel of Silk, the firſt* 40 *Ells, the ſecond* 90 : *The firſt ſells for a Crown* $\frac{1}{3}$ *of of an Ell more than the ſecond: When the Sale was over, they had taken between them* 42 *Crowns. How many Ells did each of them ſell for a Crown.*

Let $b=40$, $c=90$, $d=\frac{1}{3}$, $f=42$, and let $a+d=$ the Ells which A ſells for a Crown, then $a=$ B's.

	1	$a+d:1::b:\frac{b}{a+d}$ = the Price of A's
Then	2	$a:1::c:\frac{c}{a}=$ the Price of B's
and	3	$\frac{b}{a+d}+\frac{c}{a}=f$ per *Queſtion.*
$3\times a+d+a$	4	$aaf+afd-ab-ac=cd$
	5	for $fd-b-c$ ſubſtitute $n=116$
then	6	$aaf-na=cd$
$6\div f$	7	$aa-\frac{na}{f}=\frac{cd}{f}$
7 □ c	8	$aa-\frac{na}{f}+\frac{\frac{1}{4}nn}{ff}=\frac{cd}{f}+\frac{\frac{1}{4}nn}{ff}$
8 $uu+\frac{\frac{1}{2}n}{f}$	9	$a=\sqrt{\frac{cd}{f}+\frac{\frac{1}{4}nn}{ff}}+\frac{\frac{1}{2}n}{f}=2.999995$ or rather 3 So the 1ſt ſold $3\frac{1}{3}$ and the 2d 3 for one Crown.

80. *To find a Number, to the Quadruple of which if you add* 91, *the Whole ſhall be to the Square of the Number ſought, as* 3 *to* 4.

Let $b=91$, $c=3$, $d=4$, and $a=$ the Number ſought, then $4a=$ its Quadruple.

And	1	$c : d :: 4a+b : \frac{4ad+db}{c}$
then	2	$\frac{4ad+db}{c}=aa$ *per Queſtion.*
$2\times c-4ad$	3	$aac-4ad=db$
$3\div c$	4	$aa-\frac{4ad}{c}=\frac{db}{c}$
$4 \square c$	5	$aa-\frac{4ad}{c}+\frac{4dd}{cc}=\frac{db}{c}+\frac{4dd}{cc}$
$5\ uu$	6	$a-\frac{2d}{c}=\sqrt{\frac{db}{c}+\frac{4dd}{cc}}$
$6+\frac{2d}{c}$	7	$a=\sqrt{\frac{db}{c}+\frac{4dd}{cc}}+\frac{2d}{c}=14.$

81. *To find a Number, from the Double of which if you ſubſtract* 12, *the Square of the Remainder* — 1, *will be nine Times the Number ſought.*

Let $b=12$, $c=9$, and $\frac{1}{2}a=$ the Number ſought, then $a-b=$ the Double leſs 12.

And	1	$aa-2ab+bb-1=\frac{ca}{2}$
$1-bb-\frac{a}{2}$	2	$aa-2ab-\frac{ca}{2}=-bb+1$
		for $-2b-\frac{c}{2}$ ſubſtitute $n=285$.
then	3	$aa-na=-bb+1$
3 □ c	4	$aa-na+\frac{1}{4}nn=-bb+\frac{1}{4}nn$
4 uu	5	$a-\frac{1}{2}n=\sqrt{-bb+1+\frac{1}{4}nn}$
$5+\frac{1}{2}n$	6	$a=\sqrt{bb+1+\frac{1}{4}nn}+\frac{1}{2}n=22$
and	7	$\frac{a}{2}=11$ the Number ſought.

82. *To divide the Number* 19 *into two Parts, ſo that the Sum of the Squares of the Parts will be* 193.

Let $b=19$, $c=193$, $a=$ the greater and $e=$ the leſſer Number ſought.

Then	1	$a+e=b$ } *per Queſtion.*
	2	$aa+ee=c$ }
1 @	3	$aa+2ae+ee=bb$
$3-2$	4	$2ae=bb-c$
4×2	5	$4ae=2bb-2c$
$3-5$	6	$aa-2ae+ee=2c-bb$
6 uu	7	$a-e=\sqrt{2c-bb}=5$
$1+7$	8	$2a=b+5=24$
	9	$a=12$, and $21-12=9=$ the leſſer.

83. *To divide* 7 *into two Parts, ſo that the Difference of the Squares, which are made from the Triple of the leſs Part, and the Double of the greater may be* 17.

Put $b=7$, $c=17$, $a=$ the greater Number, then will $b-a=$ the leſſer.

Then	1	$9aa-18ab+9bb-4aa=c$ *per Q.*
$1-4aa$	2	$5aa-18ab+9bb=c$
$2-9bb\div 5$	3	$aa-\frac{18ab}{5}=\frac{c-9bb}{5}$
3 □ c	4	$aa-18ab+\frac{324bb}{100}=\frac{20c+144bb}{100}$
4, *uu* 2	5	$a-\frac{18b}{10}=\sqrt{\frac{20c+144bb}{100}}$
$5+\frac{18b}{10}$	6	$a=\sqrt{\frac{-20c+144bb}{100}}+\frac{18b}{10}=4$
and	9	$7-4=3=$ the leſſer.

84. *A Man buys a Piece of Linen, and by ſelling it again, he gains* 12 *Crowns* —$\frac{0}{20}$ *of what he bought it for: And finds by this Means, that he had gain'd as much for* 100 *Crowns, as the Linen coſt him. What Price was the Linen bought and ſold at?*

Let $a=$ the Price the Linen was bought at, $b=12$, and $c=100$, then $\frac{a}{10}=$his Profit.

And

And	1	$c : a :: a : \frac{10b - a}{10}$
∵	2	$aa = \frac{10bc - ca}{10}$
2×10	3	$10aa + ca = 10bc$
$3 \div 10 = c$	4	$aa + \frac{ca}{10} + \frac{cc}{400} = bc + \frac{cc}{400}$
$4 \circledcirc \div \frac{c}{20}$	5	$a = \sqrt[2]{bc + \frac{cc}{400}} - \frac{c}{20} = 30$
then		$100 : 30 :: 30 : 9 =$ his Gain. $30 + 9 = 39 =$ the Price he ſold it at.

85. *A Man buys* 18 *Ells of Cloth of different ſorts and colour, ſuppoſe Red and Black; what he bought of each coſt* 40 *Crowns: And he pays for every Ell of red Cltoh* 1 *Crown more than for the black. How many Ells of each Sort did he buy?*

Let $b = 18$, $c = 40$, $a =$ the Yards of black Cloth, then will $b - a =$ the Red, and $\frac{c}{a} =$ the Price of one Yard of Black, and $\frac{c}{b - a} =$ the Price of one Yard of the Red.

Then

Then	1	$\frac{c}{a}+1=\frac{c}{b-a}$ *per Queſtion.*
$1\times a$	2	$c+a=\frac{ac}{b-a}$
$2\times b-a$	3	$-aa+ab-ca+cb=ac$
$3+aa$ &c.	4	$aa+2ca-ab=cb$
		for $2c-b$ ſubſtitute $d=62$
then	5	$aa+da=cb$
$5 \square c$	6	$aa+da+\frac{1}{4}dd=cb+\frac{1}{4}dd$
6 *uu*	7	$a+\frac{1}{2}d=\sqrt{cb+\frac{1}{4}dd}$
$7-\frac{1}{2}d$	8	$a=\sqrt{cb+\frac{1}{4}dd}-\frac{1}{2}d=10$
and	9	$18-10=8$ the Yards of Red.

86. *A Man buys* 120 *Pounds of Pepper, and as many of Ginger; and received for a Crown one Pound of Ginger more than of Pepper. So that the whole Price of the Pepper came to ſix Crowns more than the Price of the Ginger. How many Pounds of each did he buy for a Crown?*

Let $b=120$, $c=6$, and $a=$ the Pounds of Pepper, then $a+1=$ the Pounds of Ginger.

	1	$\frac{c}{a}-\frac{c}{a+1}=b$ *per Queſtion.*
$1\times a+1$	2	$\frac{ac+c}{a}-c=ab+b$
$2\times a$	3	$ac+c-ac=aab+ab$
$3-ac$	4	$aab+ab=c$
$4\div b$	5	$aa+a=\frac{c}{b}$
$5\ \square\ c$	6	$aa+a+.25=\frac{c}{b}+.25$
6 *uu* 2	7	$a+.5=\sqrt[2]{\frac{c}{b}+.25}$
then	8	$a=\sqrt{\frac{c}{b}+.25}-.5=4=$ the Pepper, $a+1=5$ the Ginger.

87. *A Man buys* 80 *Pounds of Pepper, and* 36 *Pounds of Saffron, ſo that for eight Crowns he had* 14 *Pounds of Pepper more than he had of Saffron for* 26 *Crowns, and what he laid out amounted to* 188 *Crowns. How many Pounds of Pepper had he for* 8 *Crowns, and how many of Saffron for* 26?

Let $b=80$, $c=36$, $d=14$, $f=26$, $G=188$, $h=8$ and $a=$ the Pounds of Saffron for 26 Crowns, then will $a+d=$ the Pounds of Pepper.

Then

Then	1	$a : f :: c : \frac{fc}{a}$ the Price of the Saffron.
and	2	$a+d : b :: b : \frac{bb}{a+d}$, the Price of the Pepper.
then	3	$\frac{fc}{a} + \frac{bb}{a+d} = g$ *per Queſtion.*
$3 \times a+d, a$	4	$aag + adg - caf - abb = cfd$ for $dg - cf - bb$ ſubſtitute $n = 1056$
then	5	$aag + na = cfd$
$5 \div g \;\square\; c$	6	$aa + \frac{nn}{g} + \frac{\frac{1}{4}nn}{gg} = \frac{cfd}{g} + \frac{\frac{1}{4}nn}{gg}$
6 *uu*	7	$a + \frac{\frac{1}{2}n}{g} = \sqrt{\frac{cfd}{g} + \frac{\frac{1}{4}nn}{gg}}$
$7 - \frac{\frac{1}{2}n}{g}$	8	$a = \sqrt{\frac{cfd}{g} + \frac{\frac{1}{4}nn}{gg}} - \frac{\frac{1}{2}n}{g} = 6$
and	9	$a + d = 20$ the Pounds of Pepper.

88. *A and B between them owe* 174 *Pounds: A pays eight Pounds a Day, and B pays the firſt Day one Pound, the ſecond two, the third three, and ſo on. In how many Days will they clear the Debt, and how much did each of them owe?*

In this Queſtion there is given $1 + 8 = 9 = b$ the firſt Term, $c = 1$ the common Difference,

ference, and $d=174$ the Sum of all the Series of an Arithmetical Progreſſion, to find a, the Number of Terms.

$\overline{a-1}\times c=ac-c=$ the Difference between the 1ſt and laſt Term, to which add the firſt Term, and it will be $ac+b-c=$ the laſt Term, then $ac+2b-c=$ the Sum of the firſt and laſt Term, which being multiplied by $\frac{1}{2}a$, will be equal the Sum of all the Series.

Then	1	$\frac{aac+2ab-ac}{2}=d$ $\Big\}$ *per Q.*
1×2	2	$aac+2ab-ac=2d$
		for $2b-c$ Subſtitute $n=17$
then	3	$aa+na=2d$
$3 \square c$	4	$aa+na+\frac{1}{4}nn=2d+\frac{1}{4}nn$
$4\ uu$	5	$a+\frac{1}{2}n=\sqrt{2d+\frac{1}{4}nn}$
$5-\frac{1}{2}n$	6	$a=\sqrt{2d+\frac{1}{4}nn}-\frac{1}{2}n=12$
	7	$8a=96$ what A ow'd
And	8	$174-96=78$ is what B ow'd

89. *A certain Man intends to travel as many Days as he has Crowns: It happens that every following Day of his Journey he had as many Crowns as he had the Day before, beſides two Crowns over and above; and when he came to his Journey's End he finds he had in all* 45 *Crowns. How many Crowns had he at firſt?*

In this Queſtion there is given the Sum of a Series of Numbers in Arichmetical Progreſſion $=45=b$, their common Difference 2, and the Number of Terms and the leaſt equal, to find the firſt Term.

Let a $=$ the Number of Terms, or leaſt Term, then $a-1\times2=2a-2=$ the Difference between the two Extreams, to which add the 1ſt, and it will be $3a-2=$ the laſt Term, then proceed as in the laſt and you will have.

	1	$2aa-a=b$
1 ÷ 2	2	$aa-.5a=\frac{b}{2}$
2 □ *c*	3	$aa-5a+.0625=\frac{b}{2}+.0625$
3 *uu* 2	4	$a-.25=\sqrt{\frac{b}{2}+.0625}$
4 + .25	5	$a=\sqrt{\frac{b}{2}+.062}+.25=5$
for	6	$5+7+9+11+13=45.$

90. *A certain Traveller goes nine Miles a Day, three Days after another follows him, who the firſt Day travels* 4 *Miles, the ſecond* 5, *the third* 6, *and ſo on, gaining a Mile every Day. In what Time will he overtake the former?*

For the Days the firſt Travels it in put a, then the other will perform it in $a-3$, which as it is in Arithmetical Progreſſion it will be

	1	$\frac{aa+a-12}{2}=9a$ *per Queſtion.*
1 × 2	2	$aa+a-12=18a$
$2-18a+12$	3	$aa-17a=12$
3 □ *c*	4	$aa-17a+72.25=84.25$
4 *uu* +8.5	5	$a=\sqrt{84.75+8.5}=17.67331$
and	6	$a-3=14.67333091$ the Time in which he was overtaken.

91. *Two Travellers ſet out at the ſame Time from two Cities, the one from A and the other from B, which are 70 Miles diſtant from one another; one of them goes ſix Miles every Day, and the other two Miles the firſt Day, two and an half the ſecond, three the third, and ſo on, adding half a Mile to every Day's Journey. In what Time will they meet with one another?*

In this Queſtion as in Problem 88, there is given the firſt, common Difference, and Sum of the Series in an Arithmetical Progreſſion, therefore reduce the Miles to half Miles and proceed as there directed and you will have

	1	$\frac{aa+31a}{2}=140$
2 × 2	2	$aa+31a=280$
2 □ *c*	3	$aa+31a+240.25=280.25$
3 *uu* 2	4	$a+15.5=\sqrt{520.25}$
4 + 15.5	5	$a=\sqrt{520.25}-15.5=7.308989$ the Days required.

92. *Again, Two Travellers ſet out at the ſame Time from two Cities, the one from A and the other from B, which is* 120 *Miles diſtant from one another; the firſt goes* 5 *Miles a Day, and the other* 3 *Miles leſs than the Number of Days in which they meet. When will they meet?*

Let $a=$ the Number of Days, then the ſecond goes $a-3$ Miles every Day.

and	1	$aa+2a=120$ *per Queſtion.*
1 □ *c*	2	$aa+2a+1=121$
2 *uu*	3	$a+1=\sqrt{121}$
3 — 1	4	$a=\sqrt{121}-1=10=$ the Time in which they meet.

93. *A Poſt ſets out from A towards B, who travels* 8 *Miles a Day: After he had gone* 27 *Miles another ſets out from B to meet him, who goes every Day* $\frac{1}{20}$ *of the whole Journey or Diſtance of the Places A and B, and meets the firſt Poſt after ſo many Days as*

is $\frac{1}{20}$ *of the ſaid Diſtance. Required the Diſtance of A and B?*

Let $a =$ the Diſtance, then $\frac{a}{20} \times \frac{a}{20} = \frac{aa}{400}$ the Part which the ſecond Poſt travelled, and $\frac{8a}{20} + 27 =$ the Part which the 1ſt travelled.

Then	1	$\frac{aa}{400} + \frac{8a}{20} + 27 = a$ *per Q.*
1×400	2	$aa + 160a + 10800 = 400a$
$1 - 400a$	3	$aa - 240a = -10800$
$3 \square c$	4	$aa - 240a + 14400 = 3600$
4 *uu*	5	$a - 120 = \sqrt{3600}$
$5 + 120$	6	$a = \sqrt{3600} + 120 = 180$

94. *Two Merchants A and B go Partners, B brings* 420 *Crowns, and A receives out of his Gains* 52 *Crowns, and the Sum of both their Shares of Stock and Gains is* 854 *Crowns. How much did A bring, and how much did B receive out of the Gains?*

Let $b = 420$, $c = 52$, $d = 854$, and $a =$ the Money A put in, then $a : c :: b : \frac{cb}{a} =$ B's Gain.

and	1	$a+\frac{bc}{a}+b+c=d$ *per Q.*
1 × a	2	$aa+cb+ab+ac=ad$
2 − ad − cb	3	$aa+ab+ac-ad=-cb$
	4	for $b+c-d$ ſub $n=312$
3 □ c	5	$aa-na+\frac{1}{4}nn=-cb+\frac{1}{4}nn$
4 uu 2	6	$a-\frac{1}{2}n=\sqrt{-cb+\frac{1}{4}nn}$
	7	$a=\sqrt{-cb+\frac{1}{4}nn}+\frac{1}{2}n=312$
		312 : 52 : : 420 : 70 = B's Gain
Proof		$312+52+420+70=854$

95. *A Son aſks his Father how old he was? his Father replies thus; If you take* 4 *from my Age, the Remainder will be thrice the Number of your Years: But if you take one from your Age, half the Remainder will be the Square Root of my Age. Required the Age of the Father and Son?*

Put aa = the Father's Age, then $\frac{aa-4}{3}$ equal the Son's Age.

and	1	$\frac{aa-4}{3\times2}-\frac{1}{2}=a$ *per Queſt.*
1 × 6	2	$aa-4-3=6a$
2 + 7 — $6a$	3	$aa-6a=7$
3 □ c	4	$aa-6a+9=16$
4 uu	5	$a-3=\sqrt{16}$
5 + 3	6	$a=\sqrt{16}+3=7$ conſeq. $aa=49$
and	7	$\frac{49-4}{3}=15=$ the Son's Age.

96. *To find two Numbers, the Sum of whoſe Squares may be* 317, *and the Product, if they be multiply'd by one another,* 154.

Let $b=317$, $c=154$, a the greater, and e the leſſer Number ſought.

Then	1	$aa+ee=b$ } *per Queſtion.*
	2	$ae=c$
2×2	3	$2ae=2c$
$1+3$	4	$aa+2ae+ee=b+2c$
4 *uu*	5	$a+e=\sqrt{b+2c}=25$
$1-3$	6	$aa-2ae=ee=b-2c$
6 *uu*	7	$a-e=\sqrt{b-2c}=3$
$7+5$	8	$2a=28$
$8\div 2$	9	$a=14$
and	10	$25-14=11=e$.

97. *To find two Numbers, the Product of which may be* 108, *and the Difference of the Squares* 63.

Let $b=108$, $c=63$, $a=$ the greater Number, then will $\frac{b}{a}=$ the leſſer.

and	1	$aa-\frac{bb}{aa}=c$ *per Queſtion.*
$1\times aa$	2	$aaaa-bb=aac$
$2+bb-aac$	3	$aaaa-aac=bb$
$3 \;\square\; c$	4	$aaaa-aac+\frac{1}{4}cc=bb+\frac{1}{4}cc$
4 *uu*	5	$aa-\frac{1}{2}c=\sqrt{bb+\frac{1}{4}cc}$
$5+\frac{1}{2}c$	6	$aa=\sqrt{bb+\frac{1}{4}cc}+\frac{1}{2}c=144$
6 *uu*	7	$a=\sqrt{144}=12$
and	8	$108\div 12=9$ the leſſer Num.

98. *Two Farmers ſell two ſorts of Corn: A ſells* 6 *Buſhels; B receives in all, for his,* 20 *Crowns: Now, ſays B to A, if we add the Number of my Buſhels to the Number of your Crowns, the Sum will be but* 28: *Says A to B, and if I add the Square of my Crowns to the Square of your Buſhels, the Sum will be* 424. *How many Buſhels did B ſell, and how many Crowns did A receive?*

Let $b=28$, $c=424$, $a=$ B's Buſhels, $e=$ A's Crowns.

Then	1	$a+e=b$	per *Queſtion.*
	2	$aa+ee=c$	
1 ②	3	$aa+2ae+ee=bb$	
3—2	4	$2ae=bb-c$	
4 × 2	5	$4ae=2bb-2c$	
3—5	6	$aa-2ae+ee=2c-bb$	
6 *uu*	7	$a-e=\sqrt{2c-bb}=8$	
1+7	8	$2a=b+8=36$	
8—2	9	$a=18$ and $28-18=10=e$.	

99. *To find two Numbers, the firſt of which* + 2, *multiply'd into the ſecond* — 3, *may produce* 110; *and on the contrary the firſt* — 3, *multiply'd by the ſecond* + 2, *may produce* 80.

Let $a=$ the firſt, and $e=$ the ſecond Number.

Then

Then	1	$ae+2e-3a-6=110$	per Q.
and	2	$ae+2a-3e-6=80$	
$1-2$	3	$5e \quad 5a=30$	
$3\div 5$	4	$e-a=6$	
$4+a$	5	$e=6+a$	
1, 5	6	$aa+5a=104$	
6 □ *c*	7	$aa+5a+6.25=110.25$	
7 *uu*	8	$a+2.5=\sqrt{110.25}$	
$8-2.5$	9	$a=\sqrt{110.25}-2.5=8$	
5,	10	$8+6=14=e.$	

100. *If the* Sun *moves every Day one Degree, and the* Moon 13 *Degrees; and at a certain Time the* Sun *be at the Beginning of* Cancer, *and in* 3 *Days after, the* Moon *in the Beginniug of* Aries: *The Place of their next following Conjunction is demanded.*

Anſwer. 10 Degrees and $\frac{3}{4}$ of *Cancer.* For

$$\frac{13\times1\times90\times13+1\times3}{13\times1-1\times1}=\frac{1209.}{12\text{ D.}}$$

or 100 $\frac{3}{4}$ from *Aries* 00 Degrees; ſo will come out 10 $\frac{3}{4}$ of *Cancer.*

PART IV.

BEING A

MATHEMATICAL MISCELLANY, *&c.*

WE come now to the fourth Part, containing ſome neceſſary Rules concerning the Formation of a pleaſing and diverting *Ænigma*, and *Arithmetical Queſtions*, with Examples of each: Alſo how to obſerve the *Eclipſes*, and to know the fixed Stars; with ſome memorable Obſervations concerning *Time* and the Equation thereof, *Eaſter*, and Ship's Diſtance at Sea, *&c.*

Seeing Ænigma's have of old been the Diverſion of Princes, and Entertainment at Feaſts, and of late revived among our annual Writers of Diaries, it will not be amiſs, for the Information of their ingenius Aſſiſtants and Contributors, to exhibit ſome proper Rules for their Compoſition generally allowed of.

An *Ænigma* is then an ingenious and beautiful obſcuring the plaineſt Things; which when diſcovered, ſtrikes the Soul with Admiration; while we pleaſingly wonder, to ſee how it was poſſible to lay, as it were, a Veil before the Sun: It is an artificial repreſenting a Subject under the Shape of another, with ſo much Cunning, that hides a Thing while it diſcovers it, and perſuades us it is ſomething elſe than what it is really deſign'd for.

Firſt, That they propoſe all their *Ænigma*'s on Subjects generally known.

Secondly, That they aſcribe to no Subject a Property or Quality incoherent to, or inconſiſtent with it. As for Example: In an *Ænigma* upon *Snow*, it would be very abſurd to ſay, it diſcolours all the Surface of the Sea, or defies the Fury of *Ætnean* Flames.

Thirdly, That they propoſe not particular Qualities for general. For tho' by *Synecdoche* we ſay *Æthiops Albus*——*quia albos habet dentes*; yet it would be very improper in an *Ænigma* upon a *Black-moor*, to make him ſpeak in general Terms —— *I'm white as Down of Swans or falling Snow.*

Another Caution which may be reduced to this Rule is, That they propoſe not certain Numbers for uncertain.

Fourthly, That they have a Regard to Time. In treating on Subjects which have been, but now are not: They ought always to ſpeak in the *preterperfect Tenſe*, not in the *Preſent*. As were I to write an *Ænigma* on *Noah*'s Ark, 'twould be ridiculous I ſhould ſay, all Creatures are within its Womb. This would confine the Anſwerer's Thoughts intirely to Things *now in being*. To this Rule may alſo be added the Regard they ought to have to *Place*.

Theſe are Maxims ſo plain and common, that it were an Abſurdity, one would think, to violate any of them; and yet it were to be wiſhed ſome *Ænigmatiſts* in the *Diaries* would be tyed down to them. Whenever they are violated, 'tis certain, the Reader is impoſed upon; for in ſolving an *Ænigma*, until we can bring it to anſwer in every Particular, we cannot ſuppoſe we have found the Solution. If therefore the Propoſer impoſes a Falſity upon us, tho' never ſo ſmall, 'tis ſtill an Objection to our Anſwer, and we go on beating our Brains in endeavouring to apply to ſomething, that which, perhaps, is not applicable to any thing.

Example I.

THE charming *Phillis* once was wond'rous fair,
Each Youth's Delight, her Parents only Care,
Admir'd and lov'd by all the neighb'ring Swains,
And own'd the lovelieſt Nymph that grac'd the Plains.
Orant ſhe lov'd, of mean and low Degree,
A Servant in her Father's Family.
Her Parents knew how ſhe her Love had plac'd,
Yet blam'd her not, becauſe they ſaw 'twas chaſt;
One Hour to her each Morn they freely grant,
For private Converſe with her dear *Orant*.
Unſkill'd in Language he her Heart beguiles,
With amorous Looks, and with obliging Smiles.
All Shapes he wore, yet ne'er by her was ſeen
Without a beautious Face and lovely Mein.
WHEN thus he had her youthful Heart betray'd,
A foul Diſtemper ſeiz'd the blooming Maid;
Long Time a burning Fever rag'd within,
And riſing Puſtules ſpoil'd her lovely Skin.
WHILE thus ſhe lay, th' indulgent Mother near,
O give me Leave once more to ſee my Dear!
My dear *Orant!* She ſaid, 'twill eaſe my Pain,
If I can once behold *Orant* again:
Why muſt I now in vain with Tears implore
That Favour, which was ne'er deny'd before?
HER Mother cry'd, my Child, take no Offence,
Your Suit is now of dangerous Conſequence;
I can't permit you once to ſee your Love,
'Till Time ſhall this Diſorder quite remove.
AT length perceiving ſhe was left alone,
And her Deſire, as yet fulfill'd by none,

Up ſtarts th'impatient Maid without Delay,
And to *Orant*'s Apartment found the Way:
She found the Object of her Paſſion too,
And cry'd, *What ſays he to his* Phillis *now?*
You're foul, you're monſtrous grown (ſaid he) *therefore*
Henceforth I'll ſee that loathſom Face no more.
IMPATIENT of the Wrong, ſhe turn'd aſide,
And ſnatch'd a Dagger which by Chance ſhe ſpy'd,
Then, half diſtracted, to the Scornor flew,
And at his Breaſt the pointed Weapon threw:
I'll make thee feel, ungrateful Wretch, ſhe ſaid,
The juſt Reſentment of an injur'd Maid.
Pierc'd thro' and thro', he fell upon the Ground,
While ſhe, without Remorſe purſu'd the Wound:
With unrelenting Rage ſhe trampled o'er
His Body, which with utmoſt Rage ſhe tore,
And ſtrow'd his mangled Limbs upon the Floor.
LADIES, *no doubt this ſeems a monſtrous Tale,*
But if I ſhould the Myſtery reveal,
You'll own at leaſt the Story may be true,
And has been acted o'er by ſome of you.

Anſwer. *A Lady beholding her Face in a* Looking-Glaſs, *after the* Small-Pox.

Example II.

FROM Shrubs and from Trees and vaſt Caverns below
And the Sweat of Mens Bodies, our Beings we owe:
But we're odd kind of Beings, and ſtrange Pranks have play'd,
Some we have delighted, and ſome made afraid.

If two of us meet, Sir, nay, if we be three,
All Things topſie-turvy we turn preſently:
But then if our Number increas'd is to four,
We ſet all Things to Rights, Sir, as they were before,
No Hands ever had we, or Colour e'er ſaw,
Nor ever us'd Croyon or Pencil to draw:
Yet we paint with ſuch delicate Colour and Shade,
And in ſuch due Proportion our Figures are made,
That we challenge *Van Dike*, and the fam'd *Angelo*,
Such excellent Pieces as ours are, to ſhow.

Anſwer. By the Help of *Teleſcope Glaſſes* may be repreſented in a *darkned Room*, (as I have often myſelf experienced) the moſt lovely and charming Colours, Proportions and Diſtances of all outward Objects, vaſtly ſurpaſſing all that ever the moſt celebrated Painters could perform. A clear Account of this moſt wonderful and glorious Experiment, I ſhall here give you from the beſt Authors, and from my own Experience.

Procure a good convex or plano-convex-Glaſs; or if you take the Object-Glaſs of a ſeven-Foot Teleſcope, there is none hardly to be preferred before it; there being ſeveral Inconveniencies in Glaſſes, that draw above or under that Diſtance.

Make choice of a Room that hath got a *North* Window (tho' an *Eaſt* or *Weſt* may do pretty well) and let it be well darkned, ſo that no Light can come in, but at the Hole where your Glaſs is to be placed:

Then make a Hole in the Shutter of the *North* Window, about an Inch (or very little more) in Diameter; and leave open the Caſement, if there be one; for there muſt be no Glaſs beyond the Hole; then faſten the Glaſs with its Center, in the Center of the Hole, by ſome ſmall Tacks or Nails, to the Shutter, and at the Diſtance your Glaſs draws, hang up a white Sheet; (or, which is all one, move the Sheet too and fro, till you find the outer Object are repreſented on it very diſtinctly) then faſten the Sheet to the Cieling with Nails: Then will, whatever is without the Hole, and oppoſite to it, be repreſented on that Sheet, with ſuch exquiſite Exactneſs, as far ſurpaſſes the utmoſt Skill of any Painter to expreſs. For if the Sun ſhine brightly on the Objects, you will have the Colour of all Things there in their natural Paint, and ſuch an admirable Proportion of Light and Shadow, as is impoſſible to be imitated by Art; and I never ſaw any thing that comes ſo near this natural Landſkip: But if the Sun do not ſhine, the Colours will be hardly viſible; and you muſt by all Means avoid the Sun's ſhining near the Hole, for then all Things will be confuſed.

And here you have not only the Repreſentation of outward Objects, but their very Motion alſo, expreſſed on your white Cloth:

If the Wind move the Trees, Plants, or Flowers without, you have it within on your lively Picture; and nothing can be more pleaſant, than to ſee how the Colours of the moving Parts will change as they do without, by their being in various Poſitions obverted to, or ſhaded from the Light. The Motion of any Birds, Flies, or other Inſects, are painted alſo in the ſame Perfection; and the exact Lineaments of any Perſon walking at a due Diſtance without the Glaſs, will be alſo expreſſed to the Life; and all their Motions, Poſtures and Geſtures, will as plainly appear on the Cloth, as they do to any one's Eye without.

And nothing is wanting to render it one of the fineſt Sights in the World, but that all Things are inverted, and the wrong End upwards; to remedy which, take a common Looking-Glaſs, of twelve or fourteen Inches ſquare, and hold it under or near the Chin with an acute Angle to your Breaſt: For if you do ſo, and look down into it, you will ſee all Things upon the Sheet inverted in the Glaſs; and ſo (in this Caſe) will be reſtored to their natural and erect Poſitions; and this Reflection alſo from the Glaſs, gives it a Glaringneſs that is very ſurprizing, and makes it look like ſome magical Proſpect; and the moving Images, like ſo many Spectrums or Phantaſms.

taſms. And no doubt but there are many Perſons, that might eaſily be impoſed upon with ſuch a Scene; and would not be perſwaded but that it was downright Conjuration.

I have (ſaith Mr. *Harris) made uſe of this Experiment to convince ſome credulous Perſons that thoſe are abuſed and impoſed upon, who* ſee Faces in the Glaſſes of ſome cheating Knaves *amongſt us, who ſet up for cunning Men, and Diſcoverers of ſtolen Goods*, &c. *and have ſatisfied them, that much more may be done by this, and ſome other optical Experiments, and that without the Help of the Devil too, than by any of the clumſy Methods uſed by theſe Vermin.*

If the Glaſs be placed in a Sphere, or Globe of Wood (having an Hole, as large as the Glaſs, bored thro' it) which like the Eye of an Animal, may be turned every way, to receive the Rays coming from all Parts of the Objects, it will be of good Advantage to the Experiment.

You may with this Glaſs alſo (as well as with a Concave one) make that little Machine called the Magical Lanthorn; by Means of which are repreſented on a Wall in the Dark, many Phantoms and terrible Apparitions, which are taken for the Effects of Magic, by thoſe that are ignorant of the Secret. The Theory of this Lanthorn is fully explained by Mr. *Molyneux*, in his excellent Book called *Dioptr. Nova.* Prop. 56. p. 183. and the Machine at large deſcribed; to which I refer my curious Reader.

And thus is one Part of the *Example* explained. And this alſo explains two more of the former Lines in theſe Words,

But we're odd kind of Beings, and ſtrange Pranks have play'd,
Some we have delighted, and ſome made afraid.

By which is meant thoſe glorious Repreſentations of Objects in a darkned Room, upon a white Cloth (or Paper) as I have before deſcribed; and the frightful Apparitions made by the Magic Lanthorn juſt now mentioned. As for the two firſt Lines,

From Shrubs and from Trees and vaſt Caverns below,
And the Sweat of Mens Bodies, our Beings we owe;

Is meant the Materials by which Glaſs is made; which is from the Shrub called *Kali*, and *Wood-Aſhes*, and divers other Things, which, with Coals drawn out of the Bowels of the Earth, are melted; and is the hotteſt Work in the World, in Blowing and Grinding, cauſing the *Operators* to ſweat in a plentiful Manner.

SECTION II.

SECTION II.

Some uſeful and neceſſary Hints for the pleaſing Compoſition of proper Arithmetical Queſtions; by Perſons of a good mathematical Genius.

1. THE moſt natural Method in anſwering the Queſtions, ſhould be a little regarded in the Compoſition.

2. No Queſtion that can be anſwered two ways ſhould be allowed of, which will neceſſarily lead ſome from the expected Anſwer.

3. Nothing that is very paradoxical, much leſs naturally impoſſible, and void of Demonſtration, ſhould be put for an *Arithmetical Queſtion.*

4. No ambiguous or doubtful Words ſhould be uſed, but ſuch as are plain and eaſy to be underſtood.

5. We ſhould uſe the ſame Terms of Art with the lateſt and beſt Authors; or at leaſt our Terms and Phraſes ſhould be ſo plain and eaſy, as that none may be miſtaken or deceived by them: For I think it is much better and more improving, to invent ſomething ingenious than abſtruſe. Nor do I ſee any great Benefit or Difficulty in puzling Mankind, and racking their Brains; nor is it neceſſary when there is ſuch

an infinite Variety in the Mathematicks of pleaſant and profitable Propoſitions. All kind of Learning is difficult enough, eſpecially the Mathematical, to take up the ſhort Interims of our Time, without being made more tedious and laborious; therefore we ſhould make it our chief Aim to facilitate and abridge it.

Example I.

WHEN fleecy Skies had cloath'd the Ground,
With a white Mantle all around,
Then with a Greyhound, ſnowy fair,
In Milk-white Fields we cours'd a Hare;
Juſt in the midſt of a Campaign,
We ſet her up, away ſhe ran:
The Hound (I think) from her was then,
Juſt thirty Leaps (or three Times ten)
Oh! it was pleaſant for to ſee,
How th' Hare did run ſo timorouſly,
But yet ſo very ſwift that I,
Did think ſhe did not run, but fly:
When th' Dog was almoſt at her Heels,
She quickly turn'd, and down the Fields
She ran again, with full Carreer,
And 'gain ſhe turn'd to th' Place ſhe were;
At ev'ry Turn ſhe gain'd of Ground
As many Yards as the Greyhound
Could leap at thrice, and ſhe did make
Juſt ſix (if I do not miſtake)

Four times ſhe leapt for the Dog's three,
But two of the Dog's Leaps agree
With three of hers. Now pray declare,
How many Leaps he took to catch the Hare.

Anſwer.

LAST *Winter* when the Fields were cloath'd,
And to the Milk-white Snow betroth'd,
When Trees were cropt by froſty Weather,
Their Buds and Leaves lay mixt together,
Sol did in *Capricorn* appear,
And made ſhort Stay in this our Hemiſphere;
The Time, I very well remember
Was in the Middle of *December*:
A Knot of jovial Blades did meet,
And that our Mirth we might compleat,
Into the ſnowy Fields we went,
To courſe the Hare was our Intent;
We beat the Fields with wond'rous Care,
At laſt we found a boxing Hare,
Juſt in the midſt of a Campaign
We ſet her up, away ſhe ran,
With nimble Foot, o'er a large Plain.
Don't wonder I uſe Mr. *Walker*'s Verſe,
When I this courſing Match rehearſe;
For we had there the ſame Greyhound,
And with him cours'd the Hare around;
I well obſerv'd when we did find her,
The Dog was thirty Leaps behind her;
It pleas'd me ſo, methinks I ſee
The frighted Hare run timorouſly:

The Dog being almoſt at her Heels,
With Motion ſwift about ſhe wheels;
With full Carreer away ſhe ran
To th' Place where ſhe at firſt began;
Six Turns in all the Hare did make,
Their Number you did not miſtake;
Proportion due I did compare,
Between the Leaps o'th' Dog and Hare,
For her four Leaps the Dog made three,
The Sport was charming for to ſee;
Her Three, did Two of his contain,
So one in nine the Dog did gain;
How many Leaps to catch the Hare
The Dog did take, I ſhall declare,
Four Hundred and juſt Thirty-two
They were, and ſo, good Sir, adieu.

Example II.

Mr. *Richard Wrag* made a Voyage to *Conſtantinople* with Mr. *Barton*, Queen *Elizabeth*'s Ambaſſador; and during his Stay there, upon the 16th of *July*, 1594, for his Diverſion he made a Trip to the *Euxine* or *Black-Sea*; his Relation of it is thus, the 16th of *July*, accompanied with ſome other of our Nation, we went by Water to the *Black-Sea*, being 16 Miles from *Conſtantinople*; the Sea being all the Way thither, little broader than the *Thames*: Both Sides of the Shore being beautified with fair and goodly Buildings: At the Mouth of this *Boſphorus*

Boſphorus lieth a Rock ſome 80 Yards from the main Land, whereupon ſtandeth a white Marble Pillar, call'd *Pompey*'s Pillar, the Shadow whereof was 23 Foot long, at 9 of the Clock in the Forenoon (which is all he ſays of it to our Purpoſe; leaving the Heighth of the Pillar undetermined.) Now the Latitude of the Place being found to be 41 Deg. 8 Min. it is required to find the Heighth of the Pillar.

Anſwer.

The Sun's Longitude *July* 16, *Anno* 1594, at 9 of the Clock in the Forenoon, is, according to the Caroline Tables, 2 Deg. 47 Min. 31 Sec. in *Leo*. Thence,

1. *To find the Sun's Declination.*

	deg. min. ſec.
As Radius or Sine of ——	90 : 00 : 00
To the Sine of the Sun's Diſtance from the next Equinoctial Point: —— ——	57 : 12 : 29
So is the Sine of the Sun's greateſt Dec. ——	23 : 30 ; 00
To the Sine of the Suns Declination at the Day and Hour aforeſaid. ——	19 : 35 : 06

2. *To find the Sun's Heighth.*

As the Radius or Sine of ——	90 : 00 : 00
To the Contangent of the Poles Heighth —— ——	48 : 52 : 00

So is the Sine of the Sun's Diſtance from the Hour of 6. *viz.* —— —— —— 45 : 00 : 00

To the Tangent of an Arch, *viz.* 38 : 59 : 40

Which being ſubſtracted from the Complement of the Sun's Declination, ſay,

As the Coſine of the Arch found —— —— —— —— 51 : 00 : 20

To the Coſine of the remaining Arch of the Sun's Diſtance from the Pole —— —— —— 58 : 34 : 46

So is the Sine of the Poles Height —— —— 41 : 08 : 00

To the Sine of the Sun's Heighth at the Day and Hour aforeſaid. —— —— —— —— 46 : 14 : 32

3. *To find the Heighth of the Pillar.*

As Coſine of the Sun's Heighth —— —— —— —— 43 : 45 : 28

To the Length of the Shadow 23 Feet,

So is the Sine of the Sun's Heighth —— —— —— 46 : 14 : 32

To the Heighthth of the Pillar 24.0196 Feet.

The Operation of Logarithms is left to the Reader to perform at his Leiſure.

SECTION III.

By what Means we may know the Stars in the Heavens.

1. BY having the Picture or Repreſentation of the Conſtellation by you, and knowing one or more Stars of the ſame Conſtellation, by comparing the Stars in the Picture or Figure, and thoſe in the Heavens; and conſidering their Situation, Diſtances and Magnitudes in the one, you may eaſily find out thoſe in the other.

2. By the paſſing of ſome of the Planets, but eſpecially the Moon, thro' the Signs, or by ſome Eminent Stars.

3. By their coming to South, and if withal, you have their Heighth given, and then by a Quadrant find their Heighth in the Heavens, you may, with great Certainty and Eaſe find them out.

4. By their Riſing and Setting, when and on what Point of the Compaſs; or by their Azimuth and Altitude, uſing only a Quadrant and Compaſs.

5. By ſome Inſtrument as the Hemiſphere, Globes, or a Speaking-Tutor, who can point them out with Speed and Certainty.

To diſtinguiſh Planets from fixed Stars.

Planets rarely twinkle, *Jupiter* and *Venus* are bigger than fixed Stars, they ſhift their Places from a fixed Star conſiderably in a Week or two: Alſo by an *Ephemeris*; ſee what Conſtellation they are in: *Jupiter* and *Venus* are of the firſt Magnitude, *Mars* of the ſecond, *Saturn* and *Mercury* of the third: *Saturn* of a Lead Colour, *Jupiter* of a Silver, *Mars* of a Copper, *Venus* like gliſtening Silver, not far from the *Sun*, and *Mercury* like Quickſilver, ſeldom viſible, and always near the *Sun*.

SECTION IV.

Concerning ECLIPSES, *and how to obſerve them with Safety, Pleaſure and Profit.*

WHEN we conſider the Penetration and Contrivance neceſſary to lay the Foundation of Aſtronomy, we cannot but admire its firſt Inventors, ſuch as *Thales Mileſius*, who is ſaid firſt to have predicted Eclipſes: His Scholar *Anaximander*, who found out the Globous Figure of the Earth, the Equinoctial Points and Principles of Dyalling, and made the firſt Sphere or Image of the Heavens: And alſo *Pythagoras* or his Scholars, to whom we owe the Diſcovery of the beſt Syſtem of the Planets, who were probably aſſiſted by the *Chaldeans* and *Egyptains*, among whom (*Joſephus* informs us) Abraham *read Lectures of Aſtronomy and Arithmetick, which Sciences the* Egyptians *underſtood nothing of, 'til* Abraham *brought 'em from* Chaldea *into* Egypt, *and from thence they paſſed to the* Greeks. See Antiq. Book 1. Ch. 9.

But the Aſtronomers of our Age have attained to much greater Perfection in theſe Matters, eſpecially in Eclipſes of the Sun, which are much more difficult than Lunar. The Eclisſpe of the Moon has the ſame Appearance to all Spectators at the ſame Inſtant: Whereas that of the Sun may appear to one Part of the Earth totally obſcured, to another but in Part, on its *North* Side, to a third on its *South* Side, and to a fourth not at all; and all this at the ſame Moment of Time.

Hence it is, That the Lunar Eclipſes equally happening at the ſame Inſtant to that Hemiſphere to which the Moon is then viſible, may be reduced to any other Meridian, by allowing the Time, belonging to the Difference of Meridians after the Calculation is ended: But an Eclipſe of the Sun muſt be calculated for every different Meridian, if you will have its true Appearance, which much encreaſeth its Difficulty, and among other Things made the Antients to err ſo much and ſo often herein: Wherefore, after the true Conjunction is found, change the Time by the Tables into the Time proper to another Meridian, by allowing its Difference, to which Time and Place find the Parallaxes, *&c.* For the Parallax of the Moon differs in every Climate, on which the viſible Eclipſe of the Sun principally depends.

The Moon having only a precarious Light, is covered with real Darkneſs, whenever the Earth robs it of the Sun-Beams. But the Sun, on the contrary, which is luminous, or rather Light itſelf, can never be really darkned by the Moon's covering it, only in Appearance to the Spectators under its Shade; it is not the Sun but the Earth that is in Darkneſs. Hence it may more properly be term'd an Eclipſe of the Earth than of the Sun; yet becauſe of Cuſtom, and the ſeeming Darkneſs over the Sun, I retain the common Phraſe of the Eclipſe of the Sun.

In general, there are more Eclipſes of the Sun than of the Moon, but Eclipſes of the Sun in any particular Place are much fewer than of the Moon, becauſe the Moon's Shadow is leſs than the Earth's and conſequently does not involve any given Place of the Earth ſo oft as the Earth's Shadow does ſome Part of the Moon. The Moon oftner takes away the whole Sun from the

Earth, than the Earth takes away the whole Sun from the Moon; the Body of the Earth being larger, receives more Tranſits of the Moon's Shadow than the Moon of the Earth's Shadow; and becauſe the Sum of the Semidiameters of the Moon and the Earth is never leſs then 54 Minutes, and the Semidiameters of the Sun and Moon never greater than 34. The Ecliſpe of the Moon may happen in a greater Latitude of the Moon than the Solar; and in reſpect of one and the ſame Place upon the Earth, the Eclipſes of the Moon will be more frequent; tho' in reſpect of the whole Earth, Solar Eclipſes may exceed in Number.

Moreover, becauſe for many Days together, the Sun's Place is once every Year but a little diſtant from either Node, during which Time there happen generally two Syzyges; and conſequently Eclipſes which are the greater, the nearer any new or Full Moon happens to the ſaid Nodes, which Nodes are the Sections of the Moon's Orbit with the Ecliptic: In a Year there happen ſeldom leſs than two, or more than ſix Eclipſes.

Aſtronomers affirm the Sun may be eclipſed more than 12 Digits, and the Moon 22 (if it had them) for a Digit is one 12th Part of their Diameter. The Sun is ſeldom totally eclipſed in one Place, the Moon often:

And the greateſt Shadow the Moon hides, is not above 200 Miles, being the greateſt Difference of the Sun and Moon's Diameter, which according to the obliquity of Horizons is greater or leſs. The End of the Shadow, whether of the Moon or Earth, falls ſhort of the other Planets, and therefore they Eclipſe none but each other.

The Sun's Eclipſe ariſes ſooner to thoſe that inhabit the *Weſtern* Parts, and later to thoſe more *Eaſterly*, becauſe the Moon's proper Motion from *Weſt* to *Eaſt* is ſwifter than the Sun's near 13 Times; and therefore it begins on the *Weſtern* Limb of the Sun, which Part is firſt reſtored to Light again: And if the Moon's viſible Latitude be *North*, the *North* Part, but if *South*, the *South* Part of the Sun is darkned, becauſe the Moon intercepts the Light of the Sun, on the ſame Side of the Ecliptick the Moon appears.

The Sun's Eclipſe laſts not ſo long as the Moon's, the Sun's in any Place ſeldom exceeding two Hours, but the Moon's ſometimes more than four; becauſe the Moon each Hour, goes through half a Degree about the Length of the Sun's apparent Diameter, which the Moon muſt be one Hour covering, and another Hour uncovering, about two Hours in all. Solar Eclipſes alſo are very different each from other, not only becauſe of the unequal and uncertain Motion of the Moon, both true and apparent, the greater or leſſer viſible Latitude of the Moon's Diſtance from the Sun, and unequal Diſtance of both the Luminaries from the Earth; tho' Eclipſes may happen pretty near the ſame Time and Place, yet they will not, for the above Reaſons, be of the ſame Quantity and Duration; for this Rule is certain, *The Heavenly Motions are incommenſurable among themſelves nor have the ſame Phœnomena in every Reſpect, return'd alike in any Place.*

For the neweſt and exacteſt Method of calculating Eclipſes, ſee Dr. *Gregory*'s Aſtronomy, *Book* 4 *Sect.* 7. 8. and Mr. *Flamſteed*'s Doctrine of the Spehere in Sir *Jonas Moor*'s Syſtem of the Mathematicks.

Eclipſes are of manifold Uſes in Aſtronomy, Geography, Chronology, and Navigation, to correct their Tables, Maps, Globes, Account of Time, and contribute very much to the Diſcovery of Longitude both by Sea and Land; and by the Wings of Eclipſes and Parallaxes the Mind of Man flies up, and penetrates into the Celeſtial Regions: Theſe are the Charms, as ſaith the Poet, *which draw down the Sun and Moon on Earth*, or more truly the Knowledge of them to the inquiſitive Inhabitants of the Earth; nor do I know a more evident and illuſtrious

Proof of Aſtronomical Truth in Hypotheſis, or Tables, than may be had from Solar and Lunar Eclipſes.

To behold the Eclipſe without Hurt to the Eyes.

SOME ſimply prick a ſmall Hole in a Sheet of Paper, others uſe a Looking-Glaſs; but a better Way is with coloured or ſmoaked Glaſs, which defends the Eye from the Beams of the Sun, or by a Burning-Glaſs, reflecting it on a clean Sheet of Paper, held twice as far from the Paper as for Burning.

Tho' I do not here intend to preſcribe a Method to thoſe Gentlemen that are furniſhed with Aſtronomical Machines, ſuch as large Quadrants, or Sextants, whereunto are affix'd Teleſcopic or Sights, yet I ſhall humbly offer a ready Aſtronomical Apparatus for others.

Prepare a Sheet of Paper, whereon is drawn a Circle, about 6 Inches Diameter, divide its Circumference into 360, its Diameter by 6 Concentrick Circles, into 12 equal Parts for the Digits; and another Diameter for Decimal Parts into an 100, whereby the Proportion of the light and dark Parts to each other may be known; the 360 on the Limb, may ſerve to determine the Inclination of the Cuſps of the

Shade, this paſte on ſome Wall, or Board which is better.

Then Rule a Paper, like a Surveyor's Field-Book, in 9 or 10 Columns, for the Uſes following.

Obſervations common to the Eye, as Appearances of the Sky, Colour of the Eclipſe, Stars ſeen, Candles lighted, or Birds flying toward their Neſts.	*Number of Obſervations.*	*Digits and Parts.*	*Decimal Parts.*	*Time by Pendulum.*	*Time by a Sun-Dial.*	*Sun's Altitude.*	*Time corrected.*	Aſtronomical Obſervations, as Sun's Azimuth, Inclination of the Cuſps, Semidiameter of the Moon, bigger or leſs than the Sun, whether it grows bigger or leſs; whether the Moon has an Atmoſphere, eſpecially to be obſerv'd where total Darkneſs happens by red Streaks of Light before and after it; alſo how long they are viſible.

Then prepare a large Proſpect with two Glaſſes, the Eye-Glaſs Concave, the Object Convex, and affix it to a Surveying Inſtrument, or the like, with a Ball and Socket, that it may move higher or lower, or to any Point of the Compaſs; and let it ſtand at ſuch a Diſtance from the Sheet aforeſaid, as juſt to reflect the Species of the Sun to the like Diameter already thereon drawn; which let one or two mind how the Shade commences, another obſerve the Sun's Altitude by a Gnomon and End of its Shadow, whether Wall or Stake; if you have no Quadrant large enough let ſome other mind the rectified Pendulum and Sun-Dyal, and give Account of the mean and apparent

Time and Regiſter in their proper Columns; particularly noting the Beginning and End of total Darkneſs, and of the whole, when alſo each Digit is eclipſed, if the Air be clear alſo the Angle of Incidence, *&c.*

Note, The principal Things to be minded are, 1. The exact Time of the Beginning and End of the Eclipſe, and of total Darkneſs when and where ſuch Eclipſes happen. 2. The Duration of total Darkneſs. 3. The Difference of the Colour and Temper of Light. 4. The red Streaks of Light juſt preceding and following the Total Darkneſs, being Indications of an Atmoſphere about the Moon.

The Method of obſerving Eclipſes of the Moon.

Eclipſes of the Moon, are obſerved for two principal Ends; one Aſtronomical, that by comparing Obſervations with Calculations, the Theory of the Moon's Motion may be perfected, and the Tables thereof Reformed: The other Geographical, that by comparing among themſelves the Obſervations of the ſame Ecliptick Phaſes made in divers Places, the Difference of Meridians or Longitude of Places may be diſcerned.

This Knowledge of the Eclipſes, Quantity and Duration, the Shadows, Curvity and Inclination *&c.* conduce only to the former of theſe Ends. The exact Time of the Beginning, Middle, and End of the Eclipſes, as alſo in Total ones. The Beginning and End of Total Darkneſs, is uſeful for both of them.

But becauſe in Obſervations made by the bare Eye, theſe Times conſiderably differ from thoſe with a Teleſcope, and becauſe the Beginning of Eclipſes, and the End of Total Darkneſs, are ſcarce to be obſerved exactly even with Glaſſes, (none being able to diſtinguſh between the true Shadow and *Penumbra* unleſs he hath ſeen for ſome Time before, the Line ſeparating them paſs along upon the Surface of the Moon.)

Laſtly, becauſe in ſmall Partiale Eclipſes, the Beginning and End in Total ones of ſmall Continuance in the Shadow, the Beginning and End of Total Darkneſs are unfit for nice Obſervation, by Reaſon of the ſlow Change of Appearances which the oblique Motion of the Shadow then cauſeth. For theſe Reaſons, the following Method is neceſſary to accompliſh the Geographical Ends in obſerving Lunar Eclipſes, free (as far as is poſſible) from the forementioned Inconveniencies.

Firſt, it ſhall not be practicable without a Teleſcope. Secondly, the Obſerver ſhall always have Opportunity, before his principal Obſervation, to note the Diſtinction between the true Shadow and *Penumbra.* And Thirdly, it ſhall be applicable to thoſe Seaſons of the Eclipſe, when there is the ſuddeneſt Alteration of the Appearances for theſe Intents. Let there be of the eminenteſt Spots diſperſed over all Quarters of the Moon's Surface, a ſelect Number generally agreed on, to be conſtantly made uſe of to this Purpoſe, in all the Parts of the World; as for Example, thoſe which Mr. *Hevelius* calleth *M. Sinai*, *M. Ætna*, *M. Porphyrites*, *M. Serorum*, *Inſ. Beſbica*, *Inſ. Creta*, *Palus Mæotis*, *Palus Maræotis*, *Lacus Niger Major*, Let in each Eclipſe, not all, but (for Inſtance) three of theſe Spots which lie neareſt to the Ecliptick be exactly obſerved, when they are firſt touched by the true Shadow, and again when they are completely entred into it, alſo in the Decreaſe of the Eclipſe, when they are firſt fully clear from the true Shadow. For the accurate Determination of which Moments of Time, (that being in this Buſineſs of main Importance) let there be taken Altitudes of remarkable fixed Stars on this Side the Line, of ſuch as lie between the Equator and Tropic of *Cancer*; but beyond the Line of ſuch as are ſituate towards the other Tropic, and in all Places of ſuch as at the Time of Obſervation, are about four Hours diſtant from the Meridian.

To obſerve a Solar Eclipſe.

Firſt prepare a Room well darkned, into which caſt the Sun's Rays through a Teleſcope of a competent Length, as aforeſaid, to the End of which fix a Paper, paſted on a ſmall Board, at ſuch a Diſtance that the Species of the Sun may appear thereon about five or ſix Inches over; then divide both his Periphery into three hundred and ſixty Degrees, *&c.* For the better obſerving the Inclination of the Cuſps of each Phaſis, and its Diameter into Digits and Parts, by concentrick Circles for the meaſuring the obſcured Parts. Croſs thoſe Circles at right Angles to which hang a Line and Plummet by which you may keep the vertical and horizontal-Circles in their due Situation.

There is a certain Period of Eclipſes, called Dr. *Halley* the *Chaldean Saros*, which, in Leap-Years is eighteen Years, eleven Days, ſeven Hours, forty three Minutes and fifteen Seconds; but in a common Year is eighteen Years, ten Days, ſeven Hours, forty three Minutes, fifteen Seconds. This Period may ſerve very well for common Uſe to examine Eclipſes by, but not to truſt to for the preciſe Time.

Its Uſe, by Examples from Black-Monday, *which was*

Y.		D.	H.	
1652	*March*	28.	22	
18		10	7	
1670	*April*	8	5	
18		11	7	*becauſe Leap-Year*
1688	*April*	19	12	
18		10	7	
1706	*April*	29	19	
18		11	7	*becauſe Leap-Year*
1724	*May*	11	2	
18		10	7	
1742	*May*	21	9	
18		11	7	*becauſe Leap-Year*
1760	*June*	14	16	

And from the ſame to move backwards.

Y.		D.	H.
1652	*March*	18	22
18		10	7
1634	*March*	18	15
18		10	7
1616	*March*	8	8
18		11	7
1598	*Feb.*	25	1

On which Day about Noon happened ſuch another dark Day; and therefore about an Age ago was denominated *Black Saturday.*

Note. In all theſe Days paſt were great Eclipſes of the Sun, either viſible or inviſible. I will cloſe this *SECTION* with a Recapitulation of the Difference between an Eclipſe

of the Moon, which is	and Eclipſe of the Sun, which is
1. A true Obſcuration or Defect of the Moon.	1. A true Obſcuration, not of the Sun, but of the Earth.
2. 'Tis from the Shade or Penumbra of the Earth inter-	2. 'Tis from the Shade or Penumbra of the Moon interpos'd

pos'd betwixt Moon and Sun	poſed betwixt the Sun and Earth.
3. It can only be ſeen by Night, or near it.	3. 'Tis only ſeen by Day, or near it.
4. It only happens in the true Oppoſition.	4. It only happens in the viſible Conjunction.
5. It appears in all the nocturnal Hemiſphere, every where of the ſame Magnitude and Duration.	5. It doth not appear in the whole diurnal Hemiſphere; but to ſome it is greater and longer, to ſome it appears leſs and ſhorter, to ſome it appears total or annular, to others horned, or none at all.
6. It begins in the *Eaſtern* and ends in the *Weſtern* Limb of the Moon.	6. It begins in the *Weſtern* Margin of the Sun, and ends in the *Eaſtern*.
7. If the true Latitude of the Moon be *North*, then in the Middle of the Eclipſe the obſcure Part of the Moon tends towards the *South*, if the Latitude be *South*	7. If the viſible Latitude be *North*, the obſcured Part of the Sun in the Middle of the Eclipſe tends to the *North*, if *South* towards the *South*.

it tends towards the *North*.	
8. Its greateſt Duration is when the Moon is in *Perigæo* about four Hours.	8. Its greateſt Duration is when the Sun is in *Apogæo* and Moon in *Perigæo*, about three Hours in one Place and five Hours in different Places.
9. It happens oftner than the Solar Eclipſe in the ſame Place.	9. It happens oftner than the Lunar in divers Places.
10. In the ſame Year there cannot be above three Eclipſes of the Moon.	10. In one Year there cannot happen above five Solar Eclipſes.

In half a Year's Time may happen two central Eclipſes of the Sun, or almoſt central, and then all that Year will be no Lunar Eclipſes.

In no Year will happen leſs than two or more than eight Eclipſes. The Total Eclipſes of the Sun are more than the Total Eclipſes of the Moon.

SECTION V.

A new and ready Way to calculate the Motions of the Planets, for any Time paſt or to come.

FOR *the Sun.* Add 1 Minute 56 Seconds to his Place 4 Years before the given Time, and the Sum is his exact Place for the given Time; ſo you may riſe to any Year to come, by this proportionable Addition; and likewiſe look back to any Number of Years; for the Time paſt, by deducting one Minute 56 Seconds for every fourth Year.

For the Moon. Look in an Ephemeris for twelve Years before, and if you would know the Place of the *Moon* for the *1ſt* of *January*, ſee the Place of the *Moon* the 27th of *February* 12 Years before, and add four Signs 1 Degree 5 Minutes to the *Moon*'s Place that 27th of *February*, and the Total is the Place of the *Moon* on the 1ſt of *January*, deſired, ſo ſhall you be ſure to have the *Moon*'s Place within a few Minutes (when it differs moſt from the Calculation by the Tables) but for the moſt Part it will not differ at all; and thus proceed from Day to Day out of the old Ephemeris, adding ſtill 4 *S.* 1 Degree 5 Minutes, and write it againſt the anſwering Days of the new Ephemeris ſucceſſively.

For Saturn, 1 *January* 1669, I take an Ephemeris for 29 Years before, *viz.* 1640, then compute the Diſtance of *Saturn* from the *Sun* on the laſt of *December* in the Ephemeris for this Year 1668, and having done this, I run my Eye up and down in the Ephemeris 1640, in the Beginning of *January*, or elſe in the End of *December* 1639, 'till I find *Saturn* at the ſame Diſtance from the *Sun* that he is the laſt of *December* 1668; then I conſider this daily Motion from that Day to the next, and the ſame I allow him from the laſt of *December* to the firſt of *January* 1668, and ſo from Day to Day, allowing him more or leſs for his daily Motion, according as I find the Motion of *Saturn* either increaſed or diminiſhed in the old Ephemeris at the ſame Diſtance from the Sun.

For Jupiter. Take an Ephemeris for 12 Years before, or 83 Years before, and do as for *Saturn*, conſidering the daily Motion of *Jupiter* in the old Ephemeris, at the ſame Diſtance he is that Day that you deſire to know his Place for.

For Mars. Take an Ephemeris for 79 Years and do in like manner.

For Venus. Take an Ephemeris for eight Years and do ſo.

For Mercury. Take an Ephemeris for 13 Years and do ſo.

Omne bonum melius quo communius.

Queſtionleſs, much more may be done, to which I hope the Ingenious will be prompted on by this Eſſay.

* * *

A Table of the Planets, Diſtances, Eccentricities, Revolutions, &c.

	Mean Diſtance of the Sun.	Eccentricity.	Inclination of the Orbit.		Periodical Revolution.				Motion about the Axis.		Sydereal Longitude.			Aphel. Longitude Axis Node.			Apparent Semidiameter.		Heat.
			D.	M.	D.	H.	M.	S.	H.	M.	S.	D.	M.	S.	D.	M.	S.	T.	
☿	38710	7970	6	54	87	23	15	53	—	—	7	13	48	00	15	42	5	00	6
♀	72333	517	3	24	224	16	49	24	23	00	9	5	00	1	15	16	10	30	2
●	100000	1732	66	31	365	6	8	30	23	56	8	8	20	—	—	—	—	—	1
♂	152369	14100	1	52	686	23	27	30	24	40	4	1	12	00	19	10	4	00	$\frac{4}{10}$
♃	520110	25050	1	20	4332	12	20	35	9	56	5	9	50	2	8	00	24	00	$\frac{1}{27}$
♄	953800	54709	2	30	10759	6	36	26	—	—	7	28	30	2	22	30	10	30	$\frac{1}{90}$

If one Direction, Station and Retrogradation of a Planet be compared with another Direction, Station and Retrogradation, they will be found unequal to each other, not only with reſpect to the Arches in the Ecliptic, but alſo with reſpect to their Time, and this Inequality is greater in *Mars* than in *Jupiter* and *Saturn*; then chuſing a Mean betwixt the ſaid Inequality.

	♄	♃	♂	♀ and	☿	
The direct Progreſſions of	244	284	705	542	93	*Days.*
Stationary.	8	4	2	$1\frac{1}{2}$	$\frac{1}{2}$	*Days,*
Retrogradations	136	119	75	42	22	*Days.*

	is Stationary when
♄	betwixt □ and △ Aſpects with ☉
♃	in △ Aſpect with ☉
♂	betwixt $\frac{1}{8}$ and $\frac{2}{3}$ diſtant of ☉.

SECTION VI.

Of Time, and the Diviſion thereof.

A Year is ſometimes taken from the Time of the Revolution of a Planet through the Zodiac, in which a Month is ſometimes called a Year.

Another different Year is the entire Revolution (apparently) of the fix'd Stars through the Zodiac, which they call the *Annus Magnus*. But a Year is properly that

Time which the Sun takes to run thro' the Zodiac in, and is of two Sorts, the one Aſtronomical, the other Civil: The Aſtronomical is two-fold, according to the two different Bounds of the Sun's Revolution; namely, *Sydereal* and *Tropical*, the *Sydereal* Year is the Space of Time that the Sun having departed from a ſix'd Star returns to the ſame in; and it is 365 Days, ſix Hours, and nine Minutes nearly. The *Tropical* Year is that wherein the Sun departing from one of the Cardinal Points, the equinoctial or Solſtitial returns to it again, and is ſomewhat leſs than the *Sydereal*, becauſe the Cardinal Points of the Ecliptic themſelves go backwards, and as it were meeting the Sun. This *Tropical* Year is 365 Days, 5 Hours 49 Minutes, 20 Seconds nearly, and wants about 20 Minutes of the *Sydereal*.

The Civil Year is the Space of Time that the Motion of the Sun or Moon, or both point at, received by the ſettled Cuſtom of any Nation or Country. There are three Forms of the Civil Year *viz.* Either purely *Lunar*, or purely *Solar*, or *Lunar Solar*, which is made up of both. The *Lunar* Year conſiſts of twelve Lunations, or *Synodical* Months, that are finiſhed in 354 Days, then begins again, this Year wanting near 11 Days of the *Tropical* Year, would wander through all the Seaſons of the Year in about 33 Years: This Sort of Year is uſed among the *Turks.*

There are three Sorts of *Solar* Years, or ſuch as are fitted to the Motion of the Sun alone, and the Viciſſitudes of the Seaſon depending thereon; the *Egyptian*, *Julin* and *Gregorian*; the *Egyptain* Year conſiſts of 365 Days, which they divide in 12 Months, of 30 Days each, and 5 Days to be added at the End; this Year wanting near 6 Hours of the *Solar-Tropical* Year, in 4 Years it gets near a whole Day before it, and in 1460 Years its Beginning wanders through all the Seaſons of the Year.

Julius Cæſar finding this Year to want 6 Hours, he added the Day made of them in every 4 Years between the 23d and 24th of *February.* This *Julian* Year conſiſting of 465 Days, and every 4th Year of 366 Days, is beſt fitted for *Aſtronomical Computations*, becauſe it is a Mean between the Natural or Tropical Year of 365 Days, 5 Hours, and 49 Minutes; and the *Sydereal* Year of 365 Days, 6 Hours, and 9 Minutes. This Year was held *common* among all polite Nations, from *Auguſtus* (who reſtored it when almoſt loſt) unto the Year 1582, when the *Julian Kalendar* was reformed by *GREGORY* XIII; but it is ſtill in uſe by us in *Britain* and *Ireland*, &c. Yet it muſt be confeſs'd, the Quantity of our *Julian* Year is too big, on which Account the Beginning of the Year creeps forward in regard of the Seaſons, or (which is all one) the Eqinoxes and Solſtices creep backwards, in regard of the Days of the Year. And ſince this Regreſs is about 10 Minutes, 20 Seconds, in about 133 Years it will be a Day; and conſequently from the Year of *Chriſt*, 325, wherein the Council of *Nice* was held, to the Year 1582, wherein the Pope reformed the Kalendar, namely, 1260 Years, this Regreſs was 10 Days. Hence it comes to paſs, that whereas the Vernal-Equinox happened about the 21ſt Day of *March* in the Time of the *Nicene* Council, in the Year 1582, it crept to the 11th, and this Year 1751. It's on the 9th Day, differing from the Eccleſiaſtical Vernal-Equinox (which is the 21ſt of *March*,) 12 Days, which Variation is cauſed by our *Julian* Year exceeding our *Solar* Year by 11 Minutes, *near*.

SECTION VII.

On Eaſter.

OBSERVING of late, a vaſt Difference between our Eccleſiaſtical *Canons* thereupon, and the applying the celeſtial Full *Moons* and *Equinoxes*, I ſhall take this

Opportunity to impart my Thoughts thereupon, to ſatisfy the *Curious* therein.

The *Nicene* Council, *Anno* 325, decreed *Eaſter-Day* to be that *Sunday* which falls next after the firſt Full Moon, that happens after the Vernal-Equinox, which we pretend to follow and explain in our *Rubrick*, where we deſign *Eaſter-Day to be always the firſt* Sunday *after the firſt Full-Moon, which happens next after the one and twentieth Day of* March; *and if the Full-Moon happen upon Sunday,* Eaſter-Day *is the Sunday after*, Which laſt needs Amendment, for inſtead of *the Sunday after*, I ſuppoſe it ſhould be *that very Sunday*, according to both the Tables in the *Common-Prayer*; and the reſt of it alſo needs further Illuſtration from the following *Remarks.*

1*ſt*, That the Eccleſiaſtical Vernal Equinox is ſtill fix'd to the 21*ſt* of *March*, tho' the *true*, is now about the 9th or 10th, which Variation is cauſed by our *Julian* Year exceeding our *Solar* by 11 Minutes *near.*

2*d*, That the *Paſchal* Full Moon happens ſtill on the 14th from the *Paſchal* new, *i. e.* but 13 Days compleat.

3*d*, That the Words *next after the* 21*ſt*, include that Day as certainly here, as in a few Lines after *Aſcenſion Day is ſaid to be* 40 *Days after Eaſter*, includes *Eaſter-Day.*

4*th*, That the *Paſchal* Full Moons, which we obſerve are near 5 Days later than thoſe of the Heavens, becauſe they have loſt an Hour and a half every nineteen Years, ſince that general Council.

5th, The *Paſchal* New Moon is bounded with *March* the 8*th* and *April* the 5*th*, the *Paſchal* Full Moon with *March* the 21*ſt* and *April* the 18th; and *Eaſter-Day* with *March* 22*d* and *April* 25th, on which Limits, tho' they may happen yet never can exceed them.

6th, 'Till the *Kalendar* be reformed, we are not to regard the Celeſtial *Equinoxes*, *Lunations*, or *Full Moons*, as they happen in the Heavens now-a-days. But the reputed Eccleſiaſtical *Equinoxes*, *Lunations* and Full *Moons*, as they indeed happened in the Time of the *Nicene* Council, about 14 *Centuries* ago, which we obtain by the Directions of the old *Primes*, in the Eccleſiaſtical *Kalendar*, which here follow.

The Uſe, having the Prime *and* Dominical-Letter, *to find* EASTER,

IN the firſt Column quickly ſee,
Look out the *Prime*, where e'er it be,
The third *Sunday* after, *Eaſter-Day* ſhall be.
And if the *Prime* on *Sunday* be,
Reckon that for one of the Three.

Prim.	*Mar.*	*D. L.*	*Prim.*	*April.*	*D. L.*
16	8	D		1	G
5	9	E	11	2	A
	10	F		3	B
13	11	G	19	4	C
2	12	A	8	5	D
	13	B	16	6	E
10	14	C	5	7	F
	15	D		8	G
18	16	E	13	9	A
7	17	F	2	10	B
	18	G		11	C
15	19	A	10	12	D
4	20	B		13	E
	21	C	8	14	F
12	22	D	7	15	G
1	23	E		16	A
	24	F	15	17	B
9	25	G	4	18	C
	26	A		19	D
17	27	B	12	20	E
6	28	C	1	21	F
	29	D		22	G
14	30	E	9	23	A
3	31	F		24	B
			17	25	C

Example 1ſt, Suppoſe the *Prime* 8, *Dominical-Letter* C. The *Prime* is againſt *April* the 5th, the firſt *Sunday* after is the 11th Day, the ſecond *Sunday* is the 18th Day, the third *Sunday* is the 25th, for *Eaſter-Day*.

Example 2d, Suppoſe the *Prime* 16, *Dominical-Letter* D.

The *Prime* ſtands againſt *March* the 8th on *Sunday* the ſecond *Sunday* is *March* 15th, the third *Sunday* is the 22d of *March* for *Eaſter-Day*.

But if any ignorant of the Eccleſiaſtical Full Moons, ſhall apply in their Room the celeſtial Lunations and Equinoxes, they will find the Difference ſome times not only of Days but Weeks, nay a whole Month: As in 1720, 1723, 1747, 1750, 1774, 1777. (When *Eaſter*, falls high, and the *Epact* very little or very large) and if no Reformation be, the Error will be 6 Weeks, *Anno* 2437, and after 2698 *Eaſter* will never happen according to the Decree of the *Nicene* Council.

To remedy which Inconveniency, the Legiſlature have at length corrected our Reckoning, and by an Act of Parliament, which paſs'd in the 24th Year of his preſent Majeſty's Reign, it is enacted, That in the Year 1752, eleven Days ſhall be omitted after the 2d of *September*, ſo that that which wou'd otherwiſe be the 3d ſhall be deem'd and accounted the 14th of the ſaid Month, and that for the future, in order to retain the Spring-Equinox on the ſame Day of the Month, as near as may be, the ſeveral Years of our Lord 1800, 1900 2100, 2200, 2300, or any other Hunderedth Year of our Lord, which ſhall happen in Time to come, except only every fourth Hundred Year of our Lord, whereof the Year of our Lord 2000, ſhall be the Firſt, ſhall not be eſteem'd or accounted Leap-Years, but ſhall be taken as common Years, conſiſting of 365 Days and no more, and that the Years of our Lord 2000, 2400, 2800, and every 4th 100 Year of our Lord, from the ſaid Year 2000 incluſive, and alſo all other Years, which by the preſent Supputation, are eſteem'd to be Biſſextile or Leap-Years, ſhall for the future, and in all Times to come, be eſteem'd and taken for the ſame, and to conſiſt of 366 Days, in the ſame Manner as every 4th Year now does. By the above Correction the Civil Reckoning will not vary from the True, the Space of one Day in 11000 Years, which is very inconſiderable.

SECTION VIII.

Of the Equation of Time; and to regulate Clocks and Watches.

HOW beneficial and neceſſary Pendulum Clocks and Watches are in the

Affairs of Human Life, 'tis needleſs to relate: But ſomething reſpecting their right Uſe and Mathematical underſtanding of them, may be acceptable to ſeveral.

The Clocks I here mean, are ſuch as are made by a ſkilful Workman; for a Piece ill made, may vary from the equal Time a Quarter of an Hour in a Month. Whereas a Piece, made by a judicious Hand, will not err a Minute in the ſame Time, if truly adjuſted. As to the Exactneſs of theſe long and weighty Pendulums, we are by many curious Experiments ſufficiently ſatisfied, that they are the moſt ſteady of all Artifical Motions, yet it will not point out by the Index, continually the ſame Time given by the Sun on an exact Dyal, but in a few Days we may be ſenſible of the Gain or Loſs betwixt the Dyal and Pendulum Clock; now this Diſagreement is not occaſion'd by any Defect of the Clock, but proceeds from a two-fold Inequality in the heavenly Motions; to wit, the Eccentricity of the Orbit of the Earth, and the oblique Poſition of the Equator to the Ecliptic.

As to the firſt; we may obſerve, that if the yearly Orbit were indeed circular, and had the Sun in the Center, ſo that the Diſtance of the Sun from the Earth ſhould be always the ſame, no Cauſe of this Inequality would be found in the Figure of the

Earth's Orbit: But in the Elliptical Orbit, where the Quantity of the diurnal Motion is unequal, and differ in ſome Days from what it is in others, the Thing muſt be neceſſarily otherwiſe; unequal Portions being continually added to the diurnal Revolution. As for Inſtance; in the *Aphelia*, the Earth by its ſlower Motion doth daily change its Angular Poſition to the Sun leſs than it doth in the *Perihelia*, and conſequently the Angle is leſs, which it doth then finiſh over and above its entire Revolution, to be added to that Revolution, that ſo the Solar Day may be completed; for the diurnal Motion, which as to the ſuperior Focus of the Ellipſis, is always nearly equal, is conſiderably unequal, with reſpect to the Sun near the inferior Focus; and thus an Inequality muſt needs be introduced into the Natural Days.

The other Cauſe of the beforemention'd Inequality, is the oblique Poſition of the Equator to the Ecliptic. If the Axis of the Earth were always moved parallel to the Axis of the Ecliptic, and conſequently the Plane of the Equator had coincided with that of the Ecliptic, there would be nothing in the Earth's Poſition, with reſpect to the Ecliptic, that might cauſe this Inequality. But when the Diurnal and Annual Motions are performed on very different Axes, then an Inequality of Days muſt neceſſarily take

Place. For in this, the Sun's Longitude along the Ecliptic, and the diurnal Motion along the Equator, which we call the Sun's Right Aſcenſion, are both taken into the Account; and unleſs every Arch of the Ecliptic anſwered to every equal Arch of the Equator, ſo that there were no Difference betwixt the Longitude of the Sun and his Right Aſcenſion, which Thing cannot be; there muſt moſt certainly an Inequality of Days ariſe from thence, *E. G.* Let the Longitude of the Sun be five Degrees, his right Aſcenſion then muſt not by Trigonometry, exceed four Degrees, thirty-five Minutes, and a Quarter, which wants twenty-four Minutes and three Quarters to be equal with the Sun's Longitude: This Defect of twenty-four Minutes, forty-five Seconds turned into Time, comes to one Minute and forty Seconds *fere*, which Space, tho' in itſelf may ſeem ſmall, yet when it is augmented for many Days ſucceſſively, by an almoſt equal Increaſe, will introduce in a while an Inequality too great to be neglected.

Both the Quantity and Place of theſe two Cauſes of the Inequality here mentioned, are very different in this our Age, and require to be diſtinctly calculated. Now the former of theſe Cauſes, to wit, the Eccentricity, of the Orbit, which differs wholly from the latter, remits us to the *Aphelia* and

Perihelia for an Equation of Time, which anſwers to the Quantity of that Eccentricity, and is once a Year to be added to the apparent Time, and once to be ſubſtracted from it. But the latter of theſe Cauſes which is plainly different from the former, makes an Equation of Time equal to the Difference betwixt the Sun's Longitude in the Ecliptic, and his right Aſcenſion in the Equator; and which being to be reckoned from the Equinoxes, to the Solſtices, and from the Solſtices to the Equinoxes, is twice a Year addititious, and twice ablatitious, and is almoſt one and an half of the Eccentric Equation.

The whole Equation therefore of Solar Days is not to be gained from either of theſe Cauſes ſingle or alone, but from the Combination of both, for where both the Equations are addititious, or both ablatitious, the abſolute Equation of Time ariſeth from the Sum of them, which is to be added to, or taken from the true Time, according to the Tenor and Name of both the Equations; and where one is addititious (or to be added) and the other ablatitious (or to be taken from) thence from the obtaining the mean or even Time, the Difference of them is to be added to, or taken away from the true Time, according to the different Time of the Year and Title of the greater.

From this we may plainly obſerve, that

there is a Neceſſity of this Equation of natural Days, ſeeing it is founded on true Aſtronomical Principles, and proved by correct Obſervations. I ſhall in the next Place apply it to Practice in regulating curious Clocks. Firſt, we muſt obſerve that a Pendulum Clock goes equal, that is, one twenty-four Hours at any Time of the Year, is as long as another twenty-four Hours at any other Time of the Year; and this is perpetual and conſtant, and therefore ought to differ from the apparent Time ſhewn by a Sun-Dial or other Inſtrument, as much as is the Equation of Time in Exceſs or Defect. Thus, you may ſee by the Table, that there are only four Days in the Year on which the Equation of Days ceaſes, that is, the apparent and mean Time are then the ſame, *viz. April* 4th, *June* the 6th, *Auguſt* the 21ſt, and *December* the 13th. If on any of theſe we ſet a well regulated Pendulum Clock to the apparent Time ſhewn by the Sun, on any Day afterwards, it ought to differ from the Sun ſo much as is the Equation of Time by the Table, if the Equation is to be ſubſtracted, the Pendulum ought to be ſo much ſlower than the Sun, and if the Equation is to be added, the Pendulum ought to be ſo much faſter than the Sun.

Thus, if at any Time you deſire to adjuſt your Clock, and bring it to meaſure the equal Day, you muſt either add or ſubſtract the Equation of that Day to the apparent Time given by the Sun, and ſet the Clock to it. For Example, the Year 1720, *Jan.* 1*ſt*, at Noon (*the apparent Time, or true Noon being found by a Dyal, or true Meridian Line)* in the Table of Equation, I find eight Minutes, forty-ſeven Seconds, with Title Add, wherefore I ſet my Clock eight Min. forty-ſeven Seconds paſt Twelve; and obſerve the third Day if it be nine Minutes, thirty-two Seconds faſter, then the Clock goes well; but if it goes too faſt, ſcrew down the Bob, till it gain, as in the Table (at that Time of the Year) and if it go too ſlow ſcrew up the Bob, till it agree in going with the Table. Therefore your Clock muſt always go ſo much faſter or ſlower, as the Equation in the Table, agreeing with the Time of the Year.

Thoſe Pendulum Clocks by Experience is found,
Whoſe Swing in a Minute makes ſixty Rebounds,
(As by Tryal you'll find) if you meaſure their Lengths,
Will contain juſt thirty-nine Inches two Tenths;
If ſo, then how long muſt the Pendulum be,
That ſhall make the ſame Number of Swings to agree
With the Number of Inches its Length doth contain,
In the Space of a Minute, I'd know very fain.

Anſw. According to *Ricciolus* reciprocal Pendulums are to each others Lengths as the

Square of their Vibrations in the ſame Time. Therefore Inches 52,0630399162510827I which is the Cube Root of the Product of 39.2 multiplied by the Square of 60, is the true Length of the Pendulum, Number of the whoſe Vibrations (in one Minute of Time) and Inches ſhall be equal to each other.

Divide 141120 the Product aforeſaid, by any given Length, the Quotient is the Square of the Num. of Vibrations ſought. *Or*,

Divide 141120 aforeſaid, by the Square of the Number of Vibrations given, the Quotient will be the Length of the Pendulum ſought, thereunto belonging for one Minute.

Moreover, the Length of a Pendulum vibrating Seconds at *Paris*, is three Feet eight Lines and an half; let it be required to find the Length of a Pendulum vibrating Seconds at the Equator.

Becauſe the Gravity at the Poles is to the Gravity at the Equator as 692 to 689, therefore the Decreaſe of Gravity at the Equator is $\frac{3}{392}$ Parts of the whole Gravity; but the Decreaſe of Gravity at the Equator is to its Increaſe in any other Latitude, as the Square of Radius is to the Square of the Sine of the Latitude: as 1000000 Square of Radius to 565248, Square of the Sine of the Latitude 48 Degrees 45 Minutes Latitude of *Paris*, ſo is 3 the Number which repreſents the De-

Decreaſe of Gravity at the Equator to 1695 the Number which repreſents its Increaſe at *Paris*, which added to 689, the Gravity at the Equator makes 690.695 the Gravity at *Paris*.

Then as 690.695 Gravity at *Paris*,
To 689 Gravity at the Equator,
So is 36.708 Length of a Pendulum at *Paris*,
To 36.616 Length of a Pendulum at the Equator.

The Difference $\frac{92}{1000}$ Parts of an Inch agreeing pretty near to the Obſervation of Monſieur *Richer*, who at the Iſland of *Caen* whoſe Latitude is five Degrees, found that a Pendulum there was $\frac{1}{10}$ of an Inch ſhorter than at *Paris*.

ASPIRE my Genius! Help my rhyming Muſe,
In Themes I in my native Country chuſe:
Whilſt others plow the Weaves and tread the Strands
Of diſtant Oceans, and of foreign Lands;
To fill the Mouth of Fame with ſomewhat new,
(No matter 'tis how much of it be true.)
A Pendulum ſixty-one Inches long,
By which the Time I meaſur'd (was not wrong)
Vibrated freely, while that each Stone fell
Eight Times; by which the Depth I'd have you tell:
Allowing rightly for th' Approach of Sound,
That your own Works may not themſelves confound.

Such Tales we take on Truſt, from thoſe who rove,
Tho' none give Rules by which the Truth to prove.
But this by Numbers may explained be,
By thoſe who never did the Cavern ſee:
In *Derbyſhire*, a Wonder of the *Peak*,
In *Eldon-Hole*, as Poets often ſpeak;
Whoſe Depth exactly, none could e'er deſcry,
Tho' Atheiſt *Hobbs* his utmoſt Skill did try,
Who wrote *De Mirabilibus Pecci*.
And Burleſque *Cotton* does ſtrange Tales rehearſe,
In ruſtic Words, and *Hudibraſtic* Verſe,
How he this monſterous Orifice did plumb,
But could not at the Bottom of it come,
With ſixteen Hundred Yards of Rope let looſe;
And tells a Story of a Woman's *Gooſe*:
Fabulous the one, ſo may the other be,
Erroneous too, without Philoſophy;
Extenſion of the Rope might him deceive,
And ſmall Proportion which the Plumb wou'd have
To ſuch a Length; and Part in Water drown'd,
When in this vaſt Abyſs, within the Ground.
But I the Depth have found, exactly true,
By *Gravity*, a Method ſomething new.
As heavy Bodies do accelerate,
In Spaces known firſt to our *NEWTON* Great.
Four pond'rous Stones into the Well let fall,
In meaſur'd Time, agreed in Numbers all;
A Pendulum ſixty-one Inches long,
By which the Time I meaſur'd (it was not wrong)
Vibrated freely, while that each Stone fell
Eight Times: by which the Depth I'd have you tell:
Allowing rightly for th' Approach of Sound,
That your own Works may not themſelves confound

According to the Directions before given, divide 141120 by 61, and the Square-Root of the Quotient will be 48, which is the Number of Vibrations made by that Pendulum in one Hour.

Then as 48 : 60 : : 8 : 10 the Time the Stone was ſalling, and the Sound approaching.

Now if heavy Bodies fall 16 Feet and 1 Inch the 1ſt Second *&c.* and Sound 1 move 1142 Feet in the ſame Space, then let $b=10$, $c=193$ the Inches in 16 Feet and one Inch, $d=13704$ the Inches in 1142 Feet and $a=$the Time the Stone was a ſalling.

Then	1	$b-a=$the Time the Sound was aſcending.
and	2	$aac=bd-ad$ *per Queſtion*
$2+ad$	3	$aac+ad=bd$
$3\div c$	4	$aa+\frac{ad}{c}=\frac{bd}{c}$
$4 \square c$	5	$aa+\frac{ad}{c}+\frac{1}{4}\frac{dd}{cc}=bd+{}_{4}\frac{dd}{cc}$
$5\ uu$	6	$a+\frac{1}{2}\frac{d}{c}=\sqrt[2]{bd+\frac{1}{4}\frac{dd}{cc}}$
$6-\frac{1}{2}\frac{d}{c}$	7	$a=\sqrt[2]{bd+\frac{1}{4}\frac{dd}{cc}}-\frac{1}{2}\frac{d}{c}=8,8875632$

Which being ſquar'd, and that Product multiplied by 193 gives 15244.83446936

Inches, = 1270.4028724 Feet the Depth of the Well. Proof, 15244.83446936 ÷ by 13704 = 1.1124368 = the Compliment of 8.8875632.

SECTION X.

To diſcover a Ship's Diſtance at Sea, and to know when you have croſs'd the Line or the Poles, were it poſſible.

IN ſailing near Land, if you ſpy two Lands, whoſe Diſtance you know, and they bear off you one Point diſtant aſunder, when you are athwart them, you are five Times their Diſtance from Shore, if they bear two Points, then are you two Times and an ½ their Diſtance, if three Points, then 1 and ½, if 4 Points, then the ſame Diſtance, if 5 Points, but ⅔ of that Diſtance, if 6 Points, then but ½ their Diſtance.

To know when you have croſs'd the Equator.

Obſerve if you can ſee *Charles Wain* and the Guards, or Circumpolar Stars, then is the *North* Pole elevated; or if you look towards the Equator, you ſhall ſee the Stars aſcend from the Left towards the Right, if in *North* Latitude; but from the Right towards the Left, if you be in *South* Latitude.

To know whether you are going too or from the Poles.

Seeing from the *North* Pole all Places bear *South*, and near the Pole the Compaſs is of little Uſe; then get a good Clock, mark'd with 24 Hours, and an Index which ſhall every Noon return to 24, then when under the Pole, look and ſee if the Index point at 24, then that Point where the Sun is, is directly back again from the Pole.

If 12, then towards the Sun is right forwards.

If 6, then towards the Sun is Earth due *Weſt*.

If 18, then towards the Sun is Earth due *Eaſt*.

Wherefore to go directly homeward, becauſe I came out *North* and muſt return *South*, I lay the Fly of the Compaſs ſteady before me, and the *South* Point right with the Ship's Head or Stem, then if the Clock point 24 Hours, then I ſteer the Ship directly upon the Sun.

But if the Clock point	3	Then he that doth ſteer muſt keep the Sun	*S. Weſt.*
	6		*Weſt.*
	9		*N. Weſt.*
	12		*North.*
	15		*N. Eaſt.*
	18		*Eaſt.*
	21		*S. Eaſt.*

But if you would go directly forward, then lay the *North* Point right with the Ship's Head, and when the Clock doth point 12, ſteer right upon the Sun.

FINIS.

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An Honest Man a Horse did buy,
Which was both lame & poor;
A golden Guinea was the Price,
And five good shillings more.
This Horse he fed with Corn & Hay,
Till he grew wondrous sound,
And meeting with a Chap one day,
He sold him for Three Pounds.
By which he lost half the prime Cost
One fourth o'th' keeping too;
What did his keeping stand him in?
What did he lose say you?

Let $4x$ = the Keeping then

$$4x + \overset{s}{26} = 60 + 13 + x$$

$$3x + 26 = 60 + 13 = 73$$

$$3x = 47$$

$$x = \frac{47}{3} = \overset{s}{15} \cdot\cdot \overset{d}{8}$$

$$4x = \overset{£}{3} \cdot\cdot \overset{s}{2} \cdot\cdot \overset{d}{8}$$ Cost of Keeping

$$4x + 26 = \overset{£}{4} \cdot \overset{s}{8} \cdot \overset{d}{8}$$ first Cost & Keeping

Diff^ce = excess above $\overset{£}{3} = \overset{£}{1} \cdot \overset{s}{8} \cdot \overset{d}{8}$

$= \text{loss} = \overset{s}{13} + \overset{s}{15} \cdot \overset{d}{8}$

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